How Costly is External Financing? Evidence from a Structural Estimation

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ABSTRACT

We apply simulated method of moments to a dynamic model to infer the magnitude of financing costs. The model features endogenous investment, distributions, leverage, and default. The corporation faces taxation, costly bankruptcy, and linear-quadratic equity flotation costs. For large (small) firms, estimated marginal equity flotation costs start at 5.0% (10.7%) and bankruptcy costs equal to 8.4% (15.1%) of capital. Estimated financing frictions are higher for low-dividend firms and those identified as constrained by the Cleary and Whited-Wu indexes. In simulated data, many common proxies for financing constraints actually decrease when we increase financing cost parameters.

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Corporate finance is primarily the study of financing frictions. After all, Modigliani and Miller (1958) showed that a CFO can neither create nor destroy value through his financing decisions in a world without such frictions. There is little debate about the existence of market imperfections that drive a wedge between the cost of internal and external funds, with a voluminous theoretical literature buttressing the argument that external funds are costly. However, the magnitude of financing frictions is still an open question.

Empirical researchers have employed an array of methods to gauge the magnitude of financing frictions. For example, Altinkilic and Hansen (2000) estimate underwriter fee schedules. Asquith and Mullins (1986), amongst many others, measure the indirect costs of external equity by studying announcement effects. Weiss (1990) measures the direct legal costs incurred during Chapter 11 bankruptcies. Andrade and Kaplan (1998) assess the indirect costs of financial distress in a sample of highly levered transactions that became distressed. Another set of studies, for example, Fazzari et al. (1988), attempts to gain a sense of the magnitude of financial frictions using reduced-form investment regressions.

In this paper we use observed corporate financing choices in order to infer the magnitude of financing frictions by exploiting simulated method of moments (SMM). We begin by formulating a dynamic structural model of optimal financial and investment policy for a firm facing a broad set of frictions: Corporate and personal taxation, bankruptcy costs, and linear-quadratic costs of external equity. In addition, the model embeds an agency cost of debt, as the equity-maximizing manager underinvests relative to first-best. Parameters describing the firm's production technology, profitability shocks, and financing costs represent unknowns in the structural model. Of particular interest are the four financing cost parameters, namely, bankruptcy costs as a percentage of capital and three constants in a linear-quadratic cost of external equity function. Under conditions discussed below, minimizing the distance between model-generated moments and real-world moments yields consistent estimates of the unknown parameters. Less formally, one can view the estimates as answering the following question: What magnitude of financing costs "best" explains observed financing and investment patterns?

An important step in the SMM procedure involves selecting the moments to be matched. In this paper, we use three selection criteria. First, each of the moments must be informative about the financial cost parameters we seek to estimate. For example, the mean leverage ratio is informative about bankruptcy costs, and the frequency of equity issuance is informative about the fixed costs in floating new shares. Second, the moments should involve financial ratios commonly discussed in the empirical literature, that is, we use a broad set of moments that any good model of corporate finance should be able to fit. The financing moments we match are: The first and second moments of the ratio of equity issuance to assets; the frequency of equity issuance; the mean debt-to-assets ratio; the frequency of cash holdings in excess of borrowing (i.e., negative net debt); the mean payout ratio; the variance of cash distributions; the covariance of equity issuance and investment; and the covariance of debt issuance and investment. Finally, the moments should include measures that are informative about the firm's real technology; specifically, we employ the variance of investment, the serial correlation of income, and the standard deviation of profit innovations.

We begin by fitting the model to our entire sample of Compustat firms. The typical firm behaves as if it faces an underwriter charging a fee equal to $83,410 for the first million dollars (8.3%) of gross equity proceeds, with the marginal fee having a slope of $616 per million at that point. By way of contrast, Altinkilic and Hansen (2000) estimate that the marginal underwriting fee on equity is only $51,488 for the first million (5.1%), with the marginal fee rising at a rate of only $299 per million. Our SMM parameter estimates therefore support the view that there exist large indirect costs of external equity, and that corporations are sensitive to these costs.

Estimated deadweight bankruptcy costs for the full sample are 10.4% of capital, suggestive of nontrivial indirect bankruptcy costs. By way of contrast, Weiss (1990) estimates that average direct costs of bankruptcy amount to only 2.8% of the book value of total assets. The bankruptcy cost estimate generated from the SMM procedure is consistent with Andrade and Kaplan's (1998) estimates of indirect costs of financial distress, which range from 10% to 20% of total firm value.

After estimating parameters using the full sample, we reestimate the model using sub-samples obtained by splitting the sample according to proxies for financial constraints. A firm can be judged to be more or less financially constrained according to two logically distinct metrics. The first metric is the firm's need for external funds, as measured by the ratio of first-best investment to internal resources. The second metric of financing constraints is the cost of external funds, that is, the cost the firm would incur conditional upon using external funds. The literature employs a number of constraint proxies, including firm size, dividends, the Cleary (1999) index, the Whited-Wu (2005) index, and the Kaplan-Zingales (1997) index. Splitting the sample firms according to these constraint indicators, we assess whether they identify firms with high costs of external funds. We stress that this procedure only measures the ability of a proxy to identify firms with high costs of external funds, not those with a high need for funds. Indeed, firms with high costs of funds can be expected to engage in precautionary savings in order to reduce their need for funds.

We obtain our most distinct results from splitting the sample by size: We find large differences between the cost of external funds for small and large firms. This suggests the full sample parameter estimates mask heterogeneity across firms. Large firms behave as if they face small indirect costs of external finance, and small firms behave as if they face large indirect costs of external finance. We also find that low dividend firms and those identified as constrained according to the Cleary and Whited-Wu indexes behave as if facing high costs of external funds. The results on the Kaplan-Zingales index are mixed. However, this is to be expected, because their classification scheme is based upon identifying those firms with a high need for funds.

With the structural parameter estimates from the large and small firms in hand, we use the simulated model to assess the implications of costly external funds for total firm value and investment. In particular, we draw a fixed sample of random shocks and compare the behavior of four firms: Large unconstrained, large constrained, small unconstrained, and small constrained firms. For the unconstrained firms, we set bankruptcy costs and costs of external equity to zero. The constrained firms are simulated under the parameters estimated in the SMM procedure. We find that for both large and small firms the existence of costly external funds depresses the path of investment. Of particular interest is the large negative impact of financing constraints on firm value and q ratios.

Next, we use the simulated model as a laboratory to assess the elasticity of various constraint indicators with respect to the four cost of external funds parameters. This procedure is an alternative for assessing the ability of constraint indicators to identify firms that face high costs of external funds. We find that each of the three constraint indexes, Cleary, Whited-Wu, and Kaplan-

Zingales, decrease when bankruptcy costs are increased. This suggests that the three indexes are unreliable guides to the magnitude of bankruptcy costs. The reason is simple. All three indexes use high leverage ratios as an indicator for a firm being "more constrained." If bankruptcy costs are increased, the simulated firm optimally substitutes equity for debt in its financial structure, making it appear "less constrained" according to these indicators.[1] This analysis highlights the necessity of interpreting constraint indicators with caution, given that variables such as leverage ratios and stocks of cash represent endogenous responses to the firm's financing cost conditions. Firms with high financing costs will often appear to be less constrained according to conventional metrics precisely because they save in order to avoid incurring costs. The Cleary and Whited-Wu indexes do increase with the costs of external equity, because both indexes load heavily on leverage as a constraint indicator, and because the simulated firm substitutes debt for equity when equity costs increase.

We next run standard investment regressions using data generated by the simulated model. Consistent with theory, the sensitivity of investment to average q declines with each of the four financing cost parameters. Intuitively, financing frictions make the firm less responsive to changes in the shadow value of installed capital, which is correlated with average q. We stress that this result should be interpreted with caution, however, because the q coefficient may only be a reliable guide to the cost of external funds in *simulated* data. In real-world data, measurement error in q may limit the utility of the q coefficient as a guide to the cost of external funds. (See Erickson and Whited (2000) for a discussion.)

In simulated data investment-cash flow sensitivity is declining in the costs of external equity. The intuition is as follows. Even conditioning on average q, cash flow is a proxy for investment opportunities due to the concavity of the estimated profit function. When faced with higher costs of external equity, the simulated firm invests less aggressively if hit with a positive shock. Consequently, the cash flow coefficient falls. Kaplan and Zingales (2000) and Moyen (2004) document a similar effect. We also find that the cash flow coefficient increases with bankruptcy costs. Higher bankruptcy costs cause the firm to choose less debt, reducing incentives for underinvestment. In addition, the propensity of the simulated firm to hold cash increases dramatically when bankruptcy costs increase. A firm sitting on a pool of cash invests more aggressively when hit with a positive shock, resulting in a higher cash flow coefficient.

We move next to a discussion of closely related papers. Moyen's (2004) model of financially "unconstrained" firms is closest to that presented here. Our model is more general in that it features 1) linear-quadratic costs of external equity, 2) progressive taxes on cash distributions, and 3) convex corporate taxes. However, the main difference between the two papers is the empirical focus. The objective of our paper is to use SMM to estimate financing costs. In contrast, Moyen (2004) attempts to explain the seemingly contradictory evidence in the investment-cash flow debate using an exogenously parameterized model.

Cooley and Quadrini (2001) analyze a firm that can issue defaultable debt and faces proportional costs of external equity. Their model of the debt market greatly influenced the development of the model we present here. Our model is a bit more general, allowing for corporate and personal taxation and linear-quadratic costs of external equity. Cooley and Quadrini show that existing stylized facts regarding firm growth and exit can be explained by their model when one imposes a reasonable parameterization.

Cooper and Ejarque (2003) employ indirect inference to estimate costs of external equity. There is no taxation and no debt, and the costs of external equity are linear. While Cooper and Ejarque do sketch the broad outlines of a model with corporate saving and riskless debt, no estimation is performed. They state, "The model is very difficult to estimate due to the additional state variable and the need for a fine state space." The present paper overcomes the dimensionality problem. Net worth is the only endogenous state variable. In contrast with our findings, Cooper and Ejarque estimate insignificant costs of external equity; that is, the inclusion of costs of external equity does not result in a better fit between their simulated model moments and real-world moments. A possible explanation for the difference in results is that Cooper and Ejarque attempt to match moments related to production and investment, whereas we attempt to match a broad set of financing moments in addition to moments related to production and investment.

Hennessy and Whited (2005) present a dynamic model with corporate and personal taxation, proportional costs of external equity, and credit rationing. The primary objective of that paper is to show that a rational trade-off model can be reconciled with existing capital structure "anomalies." In contrast to Hennessy and Whited (2005), the firm considered in this paper is allowed to issue defaultable debt. This substantially complicates the numerical analysis because a separate subroutine is required to solve for the debt market's equilibrium.

Leary and Roberts (2005) assume the firm's objective is to keep the leverage ratio within an exogenous band. A duration model is used to make inferences about the nature of restructuring costs. They conclude that a combination of fixed plus weakly convex costs of adjustment best explains observed hazard rates. Their results are informative about the nature of financial frictions, but leave open the question of magnitudes.

The remainder of the paper is organized as follows. Section I describes the economic environment facing the firm. Section II analyzes properties of the theoretical model. Section III describes the SMM procedure and presents estimation results for various sample splits. Section IV performs numerical comparative statics on various financial constraint indicators. Section V concludes.

I. Economic Environment

A. Operating Profits

Time is discrete and the horizon infinite. There are two control variables, the capital stock (k) and the market value of one-period debt (b). Both control variables are chosen simultaneously. The yield to maturity demanded by the lender will therefore be conditioned upon the capital stock. By way of contrast, Moyen (2006) develops a dynamic model where the manager has discretion over investment *after* the terms of the loan have been determined. Her timing assumption gives rise to a classical debt overhang problem in the sense of Myers (1977). Below, we show that the manager in our model also fails to invest at the first-best level, but the source of the distortion is different.

Capital decays exponentially at rate δ. In the model, the firm can be a borrower or lender, but not both. Therefore, negative values of b represent corporate saving in the model. A more general model would relax two assumptions. First, one could allow for debt of various maturities. Second, one could allow the firm to borrow and save simultaneously. See Acharya et al. (2005) for a model featuring simultaneous borrowing and saving.

Variables with primes denote future values, minus signs denote lagged values, and subscripts denote partial derivatives. We impose the following standard assumptions on the firm's real technology:[2]

ASSUMPTION 1: The firm's operating profits are $z\pi(k) = zk^\alpha$ where $\alpha \in (0, 1)$.

ASSUMPTION 2: The shock z takes values in the compact set $Z \equiv [\underline{z}, \overline{z}]$, $0 < \underline{z} < \overline{z} < \infty$, with its Borel subsets $\mathcal{Z}$. The Markovian transition function $Q : Z \times \mathcal{Z} \to [0, 1]$ is strictly positive, has no atoms, satisfies the Feller property, and is monotone (increasing).

B. Tax System

Fazzari et al. (1988) cite the tax system as being a potentially important factor affecting the financing hierarchy and cost of funds schedule. Our goal is to parsimoniously model the salient features of the U.S. corporate income tax.

Investors are risk neutral and the risk-free asset earns a pre-tax rate of return equal to r. The tax rate on interest income at the individual level is τ_i, implying investors use $r(1 - \tau_i)$ as their discount rate. Corporate taxable income is equal to operating profits less economic depreciation less interest expense plus interest income. Interest expense is the product of the promised yield $(\widetilde{r})$ and the amount borrowed. Loss limitations are treated as a kink in the tax schedule. The tax rate when income is positive (τ_c^+) exceeds the tax rate when income is negative (τ_c^-). That is, τ_c^- is the rebate rate provided by the government when a corporation has negative taxable income.

If the firm does not default, the corporate tax bill is

$$T^c(k', b', z, z') \equiv [\tau_c^+ \chi + \tau_c^-(1 - \chi)] * \left[z'\pi(k') - \delta k' - \widetilde{r}(k', b', z)b'\right] \tag{1}$$

where χ is an indicator function for positive taxable income. In the model, it is possible for the firm to realize negative taxable income and still find it optimal to deliver the promised debt payment, due to the existence of positive continuation value. An equilibrium bond pricing identity, derived below, is used to pin down $\widetilde{r}$. For now, it should be noted that the promised yield hinges upon variables that are observable to the lender at the time of a loan's inception, and excludes the realized shock (z'). If the corporation saves, it earns r pre-tax. Thus,

$$b' < 0 \Rightarrow \widetilde{r}(k', b', z) = r \quad \forall (k', z). \tag{2}$$

The taxation of cash distributions to shareholders is complicated by the fact that corporations pay out cash through dividends and share repurchases. Corporations should use share repurchases to disgorge cash if the marginal shareholder is a taxable individual due to the lower statutory rate historically accorded to capital gains, tax deferral advantages, and the tax-free step-up in basis at death. Green and Hollifield (2003) present a model of optimal share repurchases. In their model, the first shareholders to sell into a tender offer are those with the lowest amount of locked-in capital gains. Under the optimal strategy, the effective tax rate on capital gains is only 60% of the statutory rate.

Complete substitution of repurchases for dividends is limited by the fact that the U.S. Internal Revenue Service (IRS) prohibits replacing dividends with systematic repurchases. Given the historical reluctance of the IRS to challenge repurchase programs, the optimal plan would seem to entail a rather modest percentage of dividends. Another factor that may mitigate the substitution of repurchases for dividends is concern over Securities and Exchange Commission (SEC) prosecution for stock price manipulation. SEC Rule 10b-18 provides safe harbor for firms adhering to certain restrictions on the timing and amount of shares repurchased. Cook et al. (2003) document that most corporations conform to the SEC restrictions.

To capture these effects, we model the corporation as perceiving an increasing marginal tax rate on cash distributions. Intuitively, under an optimal distribution program small cash distributions are implemented via share repurchases. Shareholders with high basis are the first to tender, implying that the capital gains tax triggered by the repurchase is low. As the firm increases the amount distributed, there are two effects. First, the basis of the marginal tendering shareholder is reduced. Second, the firm may be inclined to increase the percentage paid out as dividends due to the IRS and SEC regulations cited above. Both effects raise the marginal tax rate on cash distributions.

At the shareholder level the total distribution tax liability, as a function of cash distributions

X, is

$$T^d(X) \equiv \int_0^X \tau_d(x)dx. \tag{3}$$

The marginal tax rate on corporate cash distributions (τ_d) is increasing in the amount distributed

$$\tau_d(x) \equiv \overline{\tau}_d * [1 - e^{-\phi x}] \tag{4}$$

where $\phi > 0$. Note that under the assumed functional form for T^d there is zero tax triggered on the first dollar distributed, while the limiting marginal tax rate reaches $\overline{\tau}_d$. It is worth noting that

$$\begin{aligned} \frac{\partial^2 T^d(X_0)}{\partial X^2} &= \overline{\tau}_d \phi e^{-\phi X_0} > 0 \quad \forall X_0 \geq 0. \\ \frac{\partial^2 T^d(X_0)}{\partial \phi \partial X} &= \overline{\tau}_d X_0 e^{-\phi X_0} > 0 \quad \forall X_0 > 0. \end{aligned} \tag{5}$$

Below, $\overline{\tau}_d$ is treated as a known parameter, while ϕ is treated as an unknown parameter. In the model, convexity of the distribution tax schedule T^d creates an incentive for the corporation to smooth cash distributions. Further, higher values of ϕ raise the marginal tax rate on cash distributions. Therefore, the variance of corporate cash distributions should be informative about the unknown distribution tax parameter ϕ.

Assumptions regarding the tax system are summarized as follows:

ASSUMPTION 3: Corporate taxes are computed according to (1), where $0 < \tau_c^- < \tau_c^+ < 1$. At the individual level, interest income is taxed at rate $\tau_i \in (0, \tau_c^+)$. The marginal tax rate on cash distributions to shareholders is determined by (4), where $\overline{\tau}_d \in (0, 1)$.

C. Costs of External Equity and Debt

The main costs of external equity discussed by Fazzari et al. (1988) are tax costs, adverse selection premia, and flotation costs. Tax costs of external equity are implicit in our parameterization of the tax system. To see this, note that the total marginal tax rate on equity income in the model is $\tau_c + (1 - \tau_c)\tau_d$. In contrast, the total marginal tax rate on corporate earnings packaged as debt is equal to τ_i. Assumption 3 ensures that for a corporation earning positive taxable income with probability one, equity is taxed more heavily than debt, because $\tau_c^+ + (1 - \tau_c^+)\tau_d > \tau_i$. However, in the model, debt becomes tax disadvantaged at the margin whenever the expected marginal corporate income tax rate is sufficiently low. This effect is consistent in form with the discussion in Graham (2000).

We do not explicitly model a setting with asymmetric information. Rather, we attempt to capture the effect of adverse selection costs, along with underwriting fees, in a reduced-form fashion. The cost of external equity function is assumed to be linear-quadratic and weakly convex. More formally,

ASSUMPTION 4: The cost of external equity is equal to Λ, where

$$\begin{aligned} \Lambda(x) &\equiv \lambda_0 + \lambda_1 x + \lambda_2 x^2 \\ \lambda_i &\geq 0 \quad i = 0, 1, 2. \end{aligned}$$

SMM is used to estimate the three parameters of the equity cost function.

The convexity assumption serves a technical purpose, ensuring that the optimal financial policy is unique for $\lambda_0 = 0$.[3] In addition, the assumption of convex costs of external equity is consistent with existing theoretical models and empirical studies. Myers and Majluf (1984) consider a firm with a single all-or-nothing investment opportunity. They show that asymmetric information increases the cost of external equity if the firm is pooled with those of lower quality. If the lemons problem is sufficiently severe, good firms find it optimal to pass up positive net present value projects. Krasker (1986) presents a generalized model of adverse selection in equity markets. He considers a setting in which the firm chooses the *scale* of new investments, taking into account rational updating of market beliefs. Krasker shows that the shadow cost of external equity is convex in the number of new shares issued. Consistent with Krasker's model, Asquith and Mullins (1986) find that the negative share price reaction to equity issuance is more pronounced for larger flotations. Altinkilic and Hansen (2000) provide detailed evidence regarding underwriter fees, finding that average costs are U-shaped due to fixed costs and increasing marginal fees for larger offerings. In addition, they find that small firms face higher flotation cost schedules.

The borrowing technology consists of a standard one-period debt contract, analogous to that derived in the costly state verification models of Townsend (1979) and Gale and Hellwig (1985). To fix ideas, it is useful to think of the firm as borrowing from a single bank. The bank faces perfect competition ex ante, so that debt is fairly priced. In order for the bank to verify net worth, it must incur a cost. If the promised payment is delivered, the bank does not verify and the original shareholders retain control. In the event of default, the bank verifies net worth. The bank then enters into renegotiations with the firm.[4] The bank has all bargaining power ex post and extracts all bilateral surplus by demanding a payment that leaves the firm indifferent between continuing or not. This is equivalent to assuming that absolute priority is obeyed in default. Within this setting, we derive the firm's endogenous default rule, analogous to the smooth-pasting condition in continuous-time models.

The bankruptcy cost function is parameterized as follows:

ASSUMPTION 5: Deadweight bankruptcy costs are $\xi(1-\delta)k'$.

SMM is used to estimate the magnitude of ξ.

Embedded in the model is an agency conflict between debt and equity that depresses investment. The ex post efficient default policy entails never defaulting because default creates deadweight losses. However, for sufficiently high debt levels, the manager, acting in the interest of shareholders, defaults for some realizations of the shock. Because a portion of the capital stock is dissipated in default, the value of installed capital is reduced. This latter effect depresses investment. The extent of the investment distortion is less severe than in a model with predetermined debt (for example, Moyen (2006)) because under our timing assumptions the firm takes into account the beneficial effect of capital investment on the required bond yield.

It is also worth noting theories that we exclude from the model. Because the driving process for shocks is exogenous, the model abstracts from the risk-shifting problem. In addition, because the manager is rational and works in the interest of current shareholders, we exclude the class of investment and financing theories that invoke cognitive biases or agency conflicts between the manager and shareholders.

II. Model

A. Equity's Problem

The variable w denotes *realized net worth*,

$$w(k', b', z, z') \equiv (1-\delta)k' + z'\pi(k') - T^c(k', b', z, z') - (1+\widetilde{r}(k', b', z))b'. \tag{6}$$

In the model, there is a single endogenous state variable $\widetilde{w}$, which denotes *revised net worth.* Revised net worth is equal to realized net worth if the firm does not default. In the event of default, the firm's net worth is reset to the lowest possible amount such that equity is just willing to continue. This amount is denoted $\underline{w}(z')$. Effectively, the bank extracts all continuation surplus from the firm, consistent with the assumption that the lender holds all ex post bargaining power in default. We discuss the precise nature of the bankruptcy negotiation process in the next subsection, where we model the debt market's equilibrium.

To clarify the discussion below it is useful to derive the firm's external funding requirement for a given desired capital stock (k'). Consider first a firm that did not default in the prior period. The direct cost of the investment is $k' - (1-\delta)k$. Liquid internal funds are equal to

$$z\pi(k) - T^c(k, b, z^-, z) - (1+\widetilde{r}(k, b, z^-))b. \tag{7}$$

The external funding requirement is equal to the investment cost less liquid internal funds, which in turn is equal to the desired capital stock less revised net worth, that is,

$$k' - (1-\delta)k - [z\pi(k) - T^c(k, b, z^-, z) - (1+\widetilde{r}(k, b, z^-))b] = k' - \widetilde{w}(k, b, z^-, z). \tag{8}$$

The external equity requirement is equal to $k' - \widetilde{w} - b'$. When this amount is negative, the distribution to shareholders is positive.

Next, consider a firm that defaulted on the prior period's debt obligation. Recall that in the event of default the lender collects the physical assets and cash within the firm and also demands a payment equal to the firm's continuation value (-$\underline{w}$). Because $\widetilde{w} = \underline{w}$ for a defaulting firm, the external funding requirement is once again equal to $k' - \widetilde{w}$. This formulation captures the idea that the defaulting firm must raise more funds than those necessary to fund the desired capital stock k'. It must come up with additional funds in order to retain control.

The construction of equilibrium proceeds in two steps. In this subsection we specify equity's problem, and in the next subsection we analyze the debt market. Consider first the feasible policy correspondence $\Gamma : Z \to K \times B$. Without loss of generality, attention can be confined to compact K. The maximum allowable capital stock $\overline{k}$ is determined by

$$\overline{z}\pi_1(\overline{k}) - \delta \equiv 0. \tag{9}$$

Because $k > \overline{k}$ is not economically profitable, let $K \equiv [0, \overline{k}]$.[5]

Under the maintained assumption that $\tau_c^+ > \tau_i$, the optimal value of b is bounded below at some finite level denoted $\underline{b} \in (-\infty, 0)$. To see this, note that for firms with positive taxable income, the after-tax return on corporate saving is below that available to the shareholder investing on his own account. As the firm's cash balance increases, the precautionary motive for retention becomes negligible and funds should be distributed. The amount of funding the firm can obtain in debt markets is finite. Intuitively, increasing the promised yield beyond a certain point *reduces* debt value as the firm defaults over a greater range of realized shocks.[6] The endogenous upper bound on debt is denoted $\overline{b}(k', z)$.

The feasible policy correspondence can be expressed as

$$\Gamma(z) \equiv \{(k', b') : k' \in K \quad \text{and} \quad b' \in [\underline{b}, \overline{b}(k', z)]\}.$$

If the realized state is z', the endogenous state variable $(\widetilde{w})$ is bounded below at $\underline{w}(z')$. This lower bound reflects the negotiation between the bank and the firm in the event of default. For that same realized state, the highest possible value of the endogenous state variable is $\overline{w}(z') \equiv w(\overline{k}, \underline{b}, z', z')$.[7] The value function for the firm's equity is defined on a compact set Ω, which represents all possible nondefault states

$$\Omega \equiv \{(\widetilde{w}, z') : \widetilde{w} \in W(z')\} \tag{10}$$

$$W(z') \equiv [\underline{w}(z'), \overline{w}(z')]. \tag{11}$$

Let Φ_d and Φ_i denote indicators for positive cash distributions and equity issuance, respectively. Let $C(\Theta)$ denote the space of all bounded and continuous functions on an arbitrary set Θ. Letting

f denote an arbitrary continuous function with domain Ω, the Bellman operator (T) corresponding to equity's problem is

$$(Tf)(\widetilde{w}, z) \equiv \max_{(k',b')\in\Gamma(z)} \Phi_d[\widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k')] - \Phi_i[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')] + \left[\frac{1}{1 + r(1 - \tau_i)}\right] \int_Z f[\widetilde{w}(k', b', z, z'), z']Q(z, dz'), \tag{12}$$

subject to

i. Γ compact, convex, continuous and nondecreasing in z,

ii. $\widetilde{r} \in C(K \times B \times Z)$,

iii. $\widetilde{w}(k', b', z, z') \equiv \max\{\underline{w}(z'), w(k', b', z, z')\}$,

iv. $\underline{w} \in C(Z)$, $\underline{w}(z') < 0 \quad \forall \quad z' \in Z$, and nonincreasing.

The second constraint states that equity faces a continuous schedule determining the promised yield demanded by the bank. The third and fourth constraints state that revised net worth is bounded below by some schedule $\underline{w}$. In the next subsection, where we analyze endogenous default, we show that $\underline{w}$ necessarily satisfies condition (*iv*). The model is then closed by constructing a debt market equilibrium, pinning down a continuous $\widetilde{r}$ function.

The following lemma will prove useful:

LEMMA 1: *The operator $T : C(\Omega) \to C(\Omega)$ defined in (12) is a contraction mapping with modulus* $[1 + r(1 - \tau_i)]^{-1}$.

Proof: See Appendix A.

Proposition 1 shows the equity value function (V) exists and Proposition 2 shows it can be determined by iterating the Bellman equation.

PROPOSITION 1: *There is a unique continuous function $V : \Omega \to \Re_+$ satisfying*

$$V(\widetilde{w}, z) = \max_{k',b'\in\Gamma(z)} \Phi_d[\widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k')] - \Phi_i[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')] + \left[\frac{1}{1 + r(1 - \tau_i)}\right] \int_Z V(\widetilde{w}(k', b', z, z'), z')Q(z, dz'). \tag{13}$$

Proof: Follows from Lemma 1 and the Contraction Mapping Theorem.

PROPOSITION 2: *For arbitrary* $v^0 \in C(\Omega)$, *the sequence*

$$v^{n+1} \equiv T(v^n)$$

converges to V, *with*

$$d_\infty(v^n, V) \leq \left[\frac{1}{1 + r(1 - \tau_i)}\right]^n * d_\infty(v^0, V).$$

Proof: Follows from Lemma 1 and the Contraction Mapping Theorem.

Propositions 3 and 4 establish some useful and intuitive properties of the value function.

PROPOSITION 3: *For each* $z \in Z$, *the equity value function* $V(\cdot, z) : W(z) \rightarrow \Re_+$ *is strictly increasing.*

Proof: See Appendix A.

PROPOSITION 4: *For each* $\widetilde{w} \in W$, *the equity value function* $V(\widetilde{w}, \cdot) : Z \rightarrow \Re_+$ *is nondecreasing.*

Proof: See Appendix A.

B. Debt Market Equilibrium

In the event of default and renegotiation, equity value is pushed down to its reservation value of zero. Equity will not default if realized net worth is positive, because a positive continuation value can then be achieved even if the promised debt payment is delivered. There is a z'-contingent critical value of realized net worth, denoted $\underline{w}(z') < 0$, such that equity is just indifferent between defaulting and delivering the promised payment. The endogenous default schedule $\underline{w}(\cdot)$ is defined implicitly by the equation

$$V[\underline{w}(z'), z'] = 0 \qquad \forall\ z' \in Z. \tag{14}$$

From Proposition 3 we know that V is strictly increasing in its first argument, revised net worth. It follows that there exists a well-defined family of inverse functions, denoted $V^{-1}(\cdot, z')$, with the property that

$$V(\widetilde{w}_0, z') = \nu_0 \Leftrightarrow V^{-1}(\nu_0, z') = \widetilde{w}_0. \tag{15}$$

The default schedule can then be defined explicitly by the endogenous default condition

$$\underline{w}(z') \equiv V^{-1}(0, z'). \tag{16}$$

Proposition 5 establishes some useful and intuitive properties of the default schedule.

PROPOSITION 5: *The default schedule* $\underline{w}$: $Z \to \Re$ *is a negative-valued, continuous, and nonincreasing function.*

Proof: If revised net worth is positive, so too is equity value, thus establishing the necessity of $\underline{w} < 0$. Because V is continuous in both arguments, the inverse function V^{-1} is also continuous, which in turn implies that $\underline{w}$ must also be continuous. Also, $\underline{w}$ is nonincreasing, because increases in z' lead to (weak) increases in V (Proposition 4), which from Proposition 3 we know must be compensated by decreases in $\widetilde{w}$ to ensure satisfaction of condition (14).

Figure 1 depicts the default decision, plotting realized net worth and the default schedule as functions of the realized shock, z'. Because $w(k', b', z, \cdot)$ is strictly increasing and $\underline{w}(\cdot)$ is nonincreasing, the two functions have a single point of intersection, denoted $z_d(k', b', z)$. For shock values on the interval $[z_d(k', b', z), \overline{z}]$ the firm delivers the promised payment. If $z' < z_d(k', b', z)$, equity prefers to default because *revised* net worth exceeds *realized* net worth.

[Figure 1 about here]

The default-inducing shock z'_d is implicitly defined by the equation

$$w(k', b', z, z'_d) = \underline{w}(z'_d). \tag{17}$$

Because operating profits are weakly positive, unlevered firms ($b' \leq 0$) never shutdown. Further, the firm is able to issue a limited amount of risk-free debt. Proposition 6 summarizes.

PROPOSITION 6: *The critical shock inducing default,* $z_d : K \times B \times Z \to Z$, *is a continuous function, decreasing in the first and third arguments, and increasing in the second argument.*

Proof: See equation (17). Continuity follows from w and $\underline{w}$ being continuous. Monotonicity in the various arguments follows from monotonicity of w.

In the event of renegotiation, the bank recovers a payment sufficient to drive net worth down to $\underline{w}(z')$. The bank's recovery in default (R) is equal to

$$R(k', z') = (1 - \xi)(1 - \delta)k' + z'\pi(k') - [\tau_c^+ \chi + \tau_c^- (1 - \chi)] * [z'\pi(k') - \delta k'] - \underline{w}(z'). \tag{18}$$

This formulation of the bank's recovery assumes that in the event of default, interest deductions on the debt obligation are disallowed. This is consistent with the U.S. tax code, where recoveries in default are treated as principal first. The term $-\underline{w}(z')$ in (18) represents the "going-concern value" extracted by the lender during the renegotiation process.

The required bond yield is determined by a zero profit condition for the bank, that is,

$$b' = \left[\frac{1}{1 + r(1 - \tau_i)}\right] \left[[1 + (1 - \tau_i)\widetilde{r}(k', b', z)]b' \int_{z_d(k',b',z)}^{\overline{z}} Q(z, dz') + \int_{\underline{z}}^{z_d(k',b',z)} R(k', z')Q(z, dz')\right]. \tag{19}$$

Holding fixed the pair (k', z), for modestly risky debt $\widetilde{r}$ must be increasing in b'. However, there are limits to how much debt the firm can raise, as it eventually reaches a debt capacity where further increases in $\widetilde{r}$ actually reduce b'. Attention is therefore confined to pairs $(\widetilde{r}, b')$ where debt value is increasing in the promised yield, because other pairs are dominated on efficiency grounds. In particular, the firm would never promise to pay a higher amount if doing so caused the lender to pay less money for the bond obligation. The required bond yield is implicitly defined by the following equation

$$\widetilde{r}(k', b', z) = \left[\frac{1}{1 - \tau_i}\right] \left[\frac{1 + r(1 - \tau_i) - \int_{\underline{z}}^{z_d(k',b',z)} [R(k', z')/b']Q(z, dz')}{\int_{z_d(k',b',z)}^{\overline{z}} Q(z, dz')} - 1\right]. \tag{20}$$

This analysis closes the model, because the bond market equilibrium is consistent with the maximization problem posited for the firm (12). Constraints (*iii*) and (*iv*) are implicit in the bond pricing equation. Equation (20) implies that the function $\widetilde{r}$ is continuous, thus satisfying (*ii*). The fact that Γ is nondecreasing follows from maintained assumption that Q is monotone (increasing). This property of the transition function ensures $\overline{b}(k', \cdot)$ is increasing in z.

C. Optimal Policies

To simplify the exposition, this subsection assumes V is concave and once differentiable.[8] Although the control policies (k', b') are chosen jointly and simultaneously, for expositional purposes it useful to view the manager as first deriving the optimal financial policy for each possible k', and then optimizing over the capital stock in the second step.

Heuristically, one can view financial optimization as proceeding in two steps. First, the manager determines optimal financing, ignoring fixed costs of external equity, that is, treating $\lambda_0 = 0$. In the second step, he determines whether the intramarginal benefits of equity issuance justify the fixed cost.

The objective function to be maximized is the right side of the Bellman equation, which is denoted as Ψ:

$$\begin{aligned}\Psi(k', b') \equiv\ & \Phi_d[\widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k')] - \Phi_i[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')] \\ & + \int_{z_d(k',b',z)}^{\overline{z}} \frac{V[w(k', b', z, z'), z']Q(z, dz')}{1 + r(1 - \tau_i)}.\end{aligned} \tag{21}$$

Applying Leibniz' rule one obtains

$$\begin{aligned}\Psi_2(k', b') =\ & \Phi_i[1 + \Lambda_1(k' - \widetilde{w} - b')] + \Phi_d[1 - \tau_d(\widetilde{w} + b' - k')] \\ & + \int_{z_d(k',b',z)}^{\overline{z}} \frac{V_1[w(k', b', z, z'), z']w_2(k', b', z, z')}{1 + r(1 - \tau_i)} Q(z, dz').\end{aligned} \tag{22}$$

Solving for w_2, it follows that at an interior optimal financial policy,

$$\Phi_i[1 + \Lambda_1(k' - \widetilde{w} - b')] + \Phi_d[1 - \tau_d(\widetilde{w} + b' - k')] = \int_{z'_d}^{\overline{z}} \frac{\left[1 + (1 - \tau_c)\left(\widetilde{r} + b'\frac{\partial \widetilde{r}}{\partial b'}\right)\right] V_1(w', z')}{1 + r(1 - \tau_i)} Q(z, dz'), \tag{23}$$

where $\tau_c \in \{\tau_c^-, \tau_c^+\}$ denotes the stochastic marginal corporate income tax rate. Equation (23) states that the firm equates the marginal cost of equity finance with the discounted shadow cost of debt service.

The first-order condition for interior optimal financing simplifies if the corporate tax schedule is linear, with $\tau_c^- = \tau_c^+$. Differentiating the bond pricing identity (19) with respect to b' it is possible to show that

$$\begin{aligned}1 + (1 - \tau_c^+)\left[\widetilde{r} + b'\left(\frac{\partial \widetilde{r}}{\partial b'}\right)\right] =\ & \frac{1 + (1 - \tau_i)r + Q_2(z, z'_d)(\partial z'_d/\partial b')[\xi(1 - \delta)k' + (\tau_c^+ - \tau_i)\widetilde{r}b']}{\Pr(z' \geq z'_d)} \\ & -(\tau_c^+ - \tau_i)\left[\widetilde{r} + b'\left(\frac{\partial \widetilde{r}}{\partial b'}\right)\right].\end{aligned} \tag{24}$$

Substituting the term above into the first-order condition for an interior optimum (23), one obtains

$$1+\Phi_i\Lambda_1(k'-\widetilde{w}-b')-\Phi_d\tau_d(\widetilde{w}+b'-k')=E\{V_1[w(k',b',z,z'),z']|z'\geq z_d'\}\times \tag{25}$$
$$\left[1+\frac{(\partial z_d'/\partial b')Q_2(z,z_d')[\xi(1-\delta)k'+(\tau_c^+-\tau_i)\widetilde{r}b']-\Pr(z'\geq z_d')(\tau_c^+-\tau_i)[\widetilde{r}+b'(\partial\widetilde{r}/\partial b')]}{1+r(1-\tau_i)}\right].$$

The left side of (25) represents the cost of equity finance. The right side of the equation represents the discounted shadow cost of servicing the interest on an additional dollar of borrowing.

The financial optimality condition (25) reduces to the traditional trade-off theory when there are no distribution taxes or costs of external equity. Under these assumptions, it follows from the envelope theorem that $V_1=1$. In this case, at an interior optimal financial policy,

$$\left[\frac{\partial z_d'}{\partial b'}\right]Q_2(z,z_d')[\xi(1-\delta)k'+(\tau_c-\tau_i)\widetilde{r}b']=\Pr(z'\geq z_d')(\tau_c-\tau_i)\left[\widetilde{r}+b'\left(\frac{\partial\widetilde{r}}{\partial b'}\right)\right]. \tag{26}$$

Intuitively, in the absence of distribution taxes and flotation costs, the optimal financing policy equates marginal tax shield benefits with marginal bankruptcy costs.[9]

Of course, for some firms the fixed costs of external equity will swamp the intramarginal gains from equity issuance. Such firms will be in an equity-inertia region, neither issuing equity nor paying dividends. For such firms, debt will be the marginal source of financing for incremental investment.

Let $b^*(k')$ denote the optimal mode of financing a given capital stock, k', and let k^* denote the optimal capital stock. We know that

$$k^*\in\arg\max_{k'}\ \Psi\left[k',b^*\left(k'\right)\right].$$

We assess alternative k' choices by writing

$$\frac{d\Psi}{dk'}=\Psi_1\left[k',b^*\left(k'\right)\right]+\Psi_2\left[k',b^*\left(k'\right)\right]\frac{\partial b^*}{\partial k'}. \tag{27}$$

For firms at an interior optimal financial policy (those paying dividends or issuing equity at the margin), $\Psi_2=0$. Applying Leibniz' rule one obtains

$$\begin{aligned}\Psi_1(k',b') &= -\Phi_i[1+\Lambda_1(k'-\widetilde{w}-b')]-\Phi_d[1-\tau_d(\widetilde{w}+b'-k')] \\ &\quad+\int_{z_d(k',b',z)}^{\overline{z}}\frac{V_1[w(k',b',z,z'),z']w_1(k',b',z,z')}{1+r(1-\tau_i)}Q(z,dz').\end{aligned} \tag{28}$$

Solving for w_1 and substituting into equation (28) we obtain the following investment optimality condition for equity-issuers and dividend-payers:

$$\Phi_i(1+\Lambda_1(k'-\widetilde{w}-b'))+\Phi_d(1-\tau_d(\widetilde{w}+b'-k')) = \int_{z'_d}^{\overline{z}} \frac{V_1(w',z')\left[1+(1-\tau_c)(z'\pi_1(k')-\delta-b'\frac{\partial\widetilde{r}}{\partial k'})\right]}{1+r(1-\tau_i)} Q(z,dz'). \tag{29}$$

The left side of (29) represents the cost of equity finance and the right side represents the value of a unit of installed capital.

For firms using debt as their marginal source of funds, $\partial b^*/\partial k' = 1$. For such firms the optimality condition for investment is

$$\frac{d\Psi}{dk'} = \int_{z'_d}^{\overline{z}} \frac{V_1(w',z')[w_1(k',b',z,z')+w_2(k',b',z,z')]}{1+r(1-\tau_i)} Q(z,dz') = 0. \tag{30}$$

Here, the investment optimality condition simplifies to

$$\int_{z'_d}^{\overline{z}} V_1(w',z')(1-\tau_c)\left[z'\pi_1(k')-\widetilde{r}-\delta-b'\left(\frac{\partial\widetilde{r}}{\partial k'}+\frac{\partial\widetilde{r}}{\partial b'}\right)\right] Q(z,dz') = 0. \tag{31}$$

III. Estimated Costs of External Funds

Because the model has no closed-form solution, we opt for an estimation technique based on simulation. Specifically, we estimate unknown parameters using SMM. This procedure chooses the parameters to minimize the distance between model-generated moments and the corresponding moments from actual data. Because the moments of the model-generated data depend on the structural parameters utilized, minimizing this distance will provide consistent estimates under the conditions discussed in Appendix B.

A. Model Calibration

In order to ensure that the tax environment facing corporations is fairly stable during the observation period, the empirical sample covers 1988 to 2001. The assumed vector of tax rates is based upon Graham (2000). Graham estimates that during his sample period from 1980 to 1994, the mean tax rate on equity income at the shareholder level was 12%. Accordingly, we set the maximum tax rate on cash distributions $\overline{\tau}_d$ to be 12%. Graham also estimates that the marginal investor in taxable bonds faced a tax rate of 28.7% from 1988 to 1992 and 29.6% starting in 1993. We set $\tau_i = 29\%$. The maximum corporate tax rate is $\tau_c^+ = 40\%$, which is close to the average combined state and federal tax rates for firms in the top federal bracket. We set $\tau_c^- = 20\%$ to approximate the effect of loss limitations. The risk-free rate is $r = 2.5\%$. We set $\delta = 15\%$, consistent with the average ratio of accounting depreciation to the capital stock in our data.

The state space for (k, b, z) is discretized in order to facilitate simulation. The shock z follows an $AR(1)$ process in logs, that is,

$$\ln\left(z'\right) = \rho \ln(z) + \varepsilon', \tag{32}$$

where $\varepsilon' \sim N\left(0, \sigma_\varepsilon^2\right)$. The parameters $\left(\rho, \sigma_\varepsilon^2\right)$ of the driving process are unknowns that must also be estimated.

In order to solve the model, we transform (32) into a discrete-state Markov chain using the method in Tauchen (1986), letting $\ln(z)$ have 15 points of support in

$$\left[-4\sigma_\varepsilon \Big/ \sqrt{1-\rho^2}\,, 4\sigma_\varepsilon \Big/ \sqrt{1-\rho^2}\right].$$

In the subsequent model simulation, the space for z is expanded to include 60 points, using interpolation to find corresponding values of V, k, b, and $\tilde{r}(\cdot)$. The capital stock, k, lies in the set

$$\left[\overline{k}, \overline{k}(1-\delta)^{1/2}, \overline{k}(1-\delta), \ldots, \overline{k}(1-\delta)^{15}\right],$$

where $\overline{k}$ is defined by (9). The state space for b has half the number of points as the state space for k. We set the maximal value equal to $(1-\tau_c^+)\,\overline{k}^{\alpha}/r$ and the minimal value equal to the opposite of the maximal value. The maximal value represents a rough guess of the value of the firm. The state spaces for k and b are sufficient for our purposes in that the optimal policy never occurs at an endpoint of either state space.

The model is solved via iteration on the Bellman equation, which produces the value function $V(\tilde{w}, z)$ and policy function $h(\widetilde{w}, z)$. The numerical solution proceeds in two steps. First, we guess $\tilde{r}(k', b', z) = r$ and solve for the value function given this guess. Second, we use the solution for the value function to identify default states and then recalculate $\tilde{r}(k', b', z)$ according to (20). We then iterate on this two-step procedure until the value function converges.

The model simulation proceeds by taking a random draw of the z shock and then computing $V(\tilde{w}, z)$ and $h(\widetilde{w}, z)$. To generate simulated data comparable to Compustat, we create $S = 6$ artificial panels, containing 20,000 independent and identically distributed firms.[10] We simulate each firm for 200 time periods and then keep the last 14, which corresponds to the time span of the Compustat sample. Dropping the first part of the series allows us to observe the firm after it has worked its way out of a possibly suboptimal starting point.

We define the following variables in order to mimic the real-world data variables.

Investment/Book Real Assets	$(k' - (1-\delta)k)/\,k$
Cash Flow/Book Real Assets	$[zk^{\alpha} - T^{c}(k, b, z^{-}, z) - (1 + \widetilde{r}(k, b, z^{-}))b]/\,k$
Tobin's q	$[V(\tilde{w}, z) + (1 + \widetilde{r}(k, b, z^{-}))b]/\,k$
Operating Income/Book Real Assets	$zk^{\alpha}/\,k$
Debt/Market Value Real Assets	$b'/\,(V(\widetilde{w}, z) + b')$
(Equity Issuance or Distributions)/Book Real Assets	$(k' - \widetilde{w} - b')/\,k$

B. Selection of Moments

We now discuss the moments that we attempt to match. This issue is important since a poor choice of moments can result in large model standard errors in finite samples or an unidentified model. Basing a choice of moments on the size of standard errors constitutes data mining. As discussed above, we choose moments that are a priori informative about both the financial cost parameters and the technological parameters we seek to estimate. Heuristically, a moment is informative about an unknown parameter if that moment is sensitive to changes in the parameter.

In order to pin down $(\lambda_0, \lambda_1, \lambda_2)$ we attempt to match the first and second moments of the ratio of equity issuance to assets, the frequency of equity issuance, the frequency of negative debt, and the covariance between equity issuance and investment. The frequency and mean of equity issuance are informative about the fixed costs of issuing equity (λ_0). If the fixed costs are large, one should see infrequent flotations of large blocks of equity. The variance of equity flotations is informative about the curvature of the equity cost function (λ_2). The frequency of negative debt is informative about the extent of the firm's precautionary motive for saving, which hinges upon all the parameters of the equity cost function. The covariance between equity issuance and investment is informative about the position of equity in the financing pecking-order, which hinges upon all the parameters of the equity cost function.[11]

The average net debt-to-assets ratio is informative about deadweight costs of default (ξ). Similarly, the covariance between leverage and investment is informative about the position of debt in the financing pecking-order, which hinges upon ξ. The payout ratio and variance of cash distributions should both be informative about the distribution tax parameter ϕ.

Finally, we utilize moments that are informative about the real side of the firm. The variance of investment should be influenced by the curvature of the profit function (α). Intuitively, the firm will be conservative in responding to shocks if the marginal product of capital declines quickly (low α). The final two moments capture the features of the driving process for z: the autoregressive parameter, ρ, and the shock standard deviation, σ_ε. The moments used to identify these parameters come from estimating a first-order panel autoregression of operating income on lagged operating income using the technique in Holtz-Eakin, Newey, and Rosen (1988). The two moments that we match from this exercise are the autoregressive parameter and the standard deviation of the regression residual.[12] The two moments are directly informative about ρ and σ_ε.

C. Estimation Results

The data are described in Appendix C. Table I contains estimation results for the full sample. The first panel compares the actual moments with those from the simulated model. Evaluating the model's success depends on the yardstick. We confine our discussion to the problem areas, which suggest natural extensions of the model. The fact that the model overshoots the variance of investment suggests the need to incorporate irreversibility into the investment cost function, as in Cooper and Haltiwanger (2005). The biggest problem area for the structural model is that it overshoots the variance of payouts by a factor of almost three. There is no obvious theoretical add-on that will fix this problem. Lintner (1956) documents the tendency of corporations to smooth dividends. *Why* they do so remains an outstanding puzzle in corporate finance.

[Table I about here]

The second panel of Table I contains point estimates of the parameters. Estimated bankruptcy costs for the full sample are 10.4% of capital, significant at the 10% level. This point estimate suggests the existence of indirect costs of financial distress. The implied costs of distress are at the low end of the range estimated by Andrade and Kaplan (1998). This result also has asset pricing implications. Gomes, Yaron, and Zhang (2003) show that even relatively modest bankruptcy costs of 5% can add 1.2% to the equity premium in a general equilibrium asset pricing model. The parameters λ_0 and λ_1 are both statistically significant at the 5% level. Although λ_2 is positive, it is statistically insignificant.

In order to place the parameter estimates in context, we contrast the implied costs of external equity with the underwriting fee schedules estimated by Altinkilic and Hansen (2000). In order to make such a comparison, we must express the costs of external equity in terms of gross proceeds raised. Let p denote gross proceeds from an equity flotation and c denote the fee charged by the underwriter. In our model, the underwriting fee schedule is implicitly defined by

$$c = \lambda_0 + \lambda_1(p - c) + \lambda_2(p - c)^2. \tag{33}$$

Application of the implicit function theorem yields

$$\begin{aligned} \frac{\partial c(p_0)}{\partial p} &= 1 - [1 + \lambda_1 + 2\lambda_2(p_0 - c_0)]^{-1} \\ \frac{\partial^2 c(p_0)}{\partial p^2} &= 2\lambda_2[1 + \lambda_1 + 2\lambda_2(p_0 - c_0)]^{-3}. \end{aligned} \tag{34}$$

Based upon the estimated parameters, the typical firm acts as if facing an underwriter charging a proportional fee equal to \$83,410 on the first million dollars of gross proceeds. Because λ_2 is positive, the marginal fee schedule is rising, with a slope equal to \$616 per million when evaluated at $p_0 = 0$. The average estimated marginal fee in our sample is \$86,109. Altinkilic and Hansen (2000) estimate that the average marginal underwriting fee on equity is only \$51,488 per million, with the marginal fee rising at a rate of only \$299 per million. Our parameter estimates are therefore consistent with the view that there are large indirect costs of external equity, and that corporations are sensitive to these costs.[13]

The results in Table I mask substantial heterogeneity in financing costs across firms. This is illustrated in Tables II and III, which report parameter estimates for small and large firms, respectively. The age and size of firms are correlated. Because less mature firms may face more severe adverse selection problems, small firms may be expected to exhibit higher costs of external funds. In addition, small firms may be subject to different real shocks than large mature firms. For this reason, splitting the sample according to firm size seems reasonable. In order to mitigate problems arising from classifying firms according to an imperfect measure of access to external finance, we discard the middle third of the sample for all of the constraint indicators discussed below.

[**Tables II and III about here**]

For small firms, the parameters λ_1 and ξ are both statistically significant at the 5% level, while λ_0 is significant at the 10% level. Small firms behave as if they face an underwriter charging a proportional fee of \$107,143 for the first million dollars (10.7%) of gross equity proceeds, with the marginal fee having a slope of \$569 per million at that point. Similarly, small firms act as if they face relatively large bankruptcy costs, equal to 15.1% of capital. Taken together, these estimates support the view that small firms face large indirect costs of external funds.

In contrast, Table III suggests that large firms do not face large indirect costs. In fact, the only statistically significant financing cost parameter for large firms is λ_1. A one-sided test of the null hypothesis that the estimates of λ_1 are equal across large and small firms is rejected at the 10% level with a probability value of 0.065. Large firms behave as if they face an underwriter charging a proportional fee of \$50,332 for the first million dollars of gross equity proceeds, with the marginal fee having a slope of \$343 per million at that point. These figures are strikingly close to those estimated by Altinkilic and Hansen (2000). This is encouraging, given that the SMM procedure infers financing costs from observed financing behavior, as opposed to employing direct measurement. Large firms also act as if they face relatively small deadweight bankruptcy costs, equal to 8.4% of capital.

The parameter estimates in Tables II and III also reveal substantial differences in the nature of the shocks hitting the two classes of firms. Large firms operate in more predictable environments, with ρ large and σ_ε small. Accounting for differences in the driving process for the shocks is therefore important to understanding why firms choose different financial policies. For example, the stable environments of the large firms account for their tendency to shun external equity finance, despite lower equity financing costs.

In order to shed light on the economic significance of the estimated financing frictions, we contrast the behavior of "constrained" and "unconstrained" firms. We draw a fixed sample of 1000 random shocks and compare the behavior of four simulated firms: Large unconstrained, large constrained, small unconstrained, and small constrained firms. All the firms are simulated using the estimated class-specific parameters describing the firm's real and tax environment: α, σ_{ε}, ρ, and ϕ. The constrained firms are simulated using their respective financing cost parameters estimated in Tables II and III. For the unconstrained firms, we set bankruptcy costs (ξ) and direct costs of external equity ($\lambda_1, \lambda_2, \lambda_3$) to zero.

The results are reported in Table IV. Beginning with the first column, note that for either size category the simulated unconstrained firms invest more aggressively than their constrained counterparts. This is consistent with the argument that financing frictions have real implications. The wedge between constrained and unconstrained investment is particularly pronounced for the small firms. This is predictable, given that the simulated small firms face high financing costs. Moving to the second column, we note that financing frictions have nontrivial implications for firm value. In particular, unconstrained firms exhibit higher Tobin's q ratios than their constrained counterparts. The difference between constrained and unconstrained firm q ratios is particularly pronounced for small firms.

[**Table IV about here**]

Financing frictions lead to predictable changes in financing behavior for the simulated firms. Bankruptcy costs cause the constrained firms to be less aggressive in exploiting debt tax shields. This difference is reflected in the relatively low debt-to-assets ratio of constrained firms. Finally, costs of external equity cause the constrained firms to issue smaller blocks of equity and to issue equity less frequently.

The baseline model contains no underwriting fees on debt flotations. As a robustness check, we assess the sensitivity of parameter estimates to this assumption. Based upon Altinkilic and

Hansen (2000), we estimate model parameters under the alternative assumption that the firm incurs underwriting fees equal to 1.09% of the gross proceeds from a debt flotation. The results are presented in Table V. The parameter estimates do not differ substantially from those presented in Tables I to III. In addition, the direction of the change is predictable. In Table V, the proportional cost of external equity (λ_1) is a bit higher than that estimated in the baseline model, while the bankruptcy cost (ξ) is a bit lower. Intuitively, the inclusion of debt flotation costs increases the attractiveness of equity and decreases the attractiveness of debt, ceteris paribus. To offset these effects, and best fit the real-world moments, the estimated costs of external equity should increase and the costs of bankruptcy should decrease.

[Table V about here]

D. Estimates Based upon Other Constraint Indicators

We reestimate the model using alternative finance constraint proxies to split the sample. The objective is to assess whether alternative constraint proxies identify firms with high costs of external funds. The literature employs a number of constraint proxies, including small firm size, low dividends, the Cleary (1999) index, the Whited-Wu (2005) index, and the Kaplan-Zingales (1997) index.[14] Dividend payout as an indicator of costly external finance is used by Fazzari et al. (1988), motivated by the idea that low dividend firms have no internal cash for financing investment and thus must look to external sources. The three indexes of financial constraints are described in detail in Appendix C. We normalize each index so that a high value indicates a more constrained firm. Briefly, the Kaplan-Zingales index isolates firms with low cash stock, low cash flow, and high debt burdens. Firms with a high Whited-Wu index are small, rely heavily on equity financing, have low cash flow, and are slow-growing firms in fast-growing industries. Firms with a high Cleary index are slow-growing, have low profit margins, and have few resources to cover their debt burdens.

In Table VI we split the sample firms according to these four constraint indicators. The small firm parameter estimates discussed above offer strong support for the common practice of using size to identify firms that are likely to face higher costs of external funds. The estimates in Table VI indicate that low dividend firms and those identified as constrained using the Whited-Wu and

Cleary indexes also behave as if facing high costs of external funds. For each of these constrained subsamples, the estimates of λ_0, λ_1, and ξ are often significant at the 5% level. The estimate of λ_2 for the high Whited-Wu firms is significant at the ten percent level. Further, a one-sided test of the null hypothesis that the estimates of λ_1 are equal across the Whited-Wu groups is rejected at the 10% level with a probability value of 0.069. In contrast, firms identified as constrained according to the Kaplan-Zingales index do not generally exhibit high costs.

[**Table VI about here**]

At this point it is worth noting that none of the three indexes need necessarily identify firms with high costs of external funds. The components of each index identify firms with a high need for funds as opposed to those with a high cost of external funds. It is easy to envision a firm with low costs of external funds that also has low internal resources relative to first-best investment. To see this, note that a firm facing low costs of external funds has little incentive to incur the tax costs associated with hoarding cash. Hence, a firm facing low costs of external finance can potentially be classified as constrained according to the three indexes.

IV. The Effect of Financing Costs on Constraint Proxies

In this section we use the simulated model as a laboratory to assess the elasticity of various constraint indicators with respect to the four cost of external funds parameters. This represents an alternative procedure for assessing the ability of constraint indicators to identify firms that face high costs of external funds. For example, if a constraint index falls when a particular cost of external finance is increased it can be argued that the constraint index is an unreliable guide to inferring the magnitude of such costs.

We evaluate the behavior of five common proxies for financing constraints: The Cleary index, the Whited-Wu index, the Kaplan-Zingales index, the sensitivity of investment to average q, and the sensitivity of investment to cash flow. We examine the sensitivity of these proxies with respect to the model parameters by calculating elasticities. The elasticity of an arbitrary variable y with respect to an arbitrary parameter κ is computed as follows. Suppose that the estimated value of the parameter κ is $\widehat{\kappa}$. We simulate the model three times, setting κ equal to $\widehat{\kappa}$, $\underline{\kappa} \equiv 0.5\widehat{\kappa}$, and $\overline{\kappa} \equiv 1.5\widehat{\kappa}$. The elasticity of y with respect to κ is computed as

$$\epsilon_{y\kappa} \equiv \frac{y(\overline{\kappa}) - y(\underline{\kappa})}{\overline{\kappa} - \underline{\kappa}} * \frac{\widehat{\kappa}}{y(\widehat{\kappa})}. \qquad (35)$$

Before discussing the properties of the constraint indicators, we first discuss some properties of the model. Table VII is intended to give the reader a sense of the causal mechanisms in the model. As the costs of external equity (λ_i) are increased, the mean and frequency of equity issuance decrease. In addition, the covariance between investment and equity issuance decreases, whereas the covariance of investment and leverage increases. These properties of the model reflect the fact that equity's position in the financial pecking-order declines when the λ parameters are increased. It is also worth noting that the variance of investment declines in the λ parameters. This suggests that high λ values cause the firm to react less aggressively to changes in the expected marginal product of capital.

[**Table VII about here**]

Table VII shows that higher bankruptcy costs (ξ) have the opposite effect. When bankruptcy costs are increased, the firm substitutes equity for debt. The mean and frequency of equity issuance increase, the covariance between investment and equity issuance increases, and the covariance of investment and leverage decreases. These properties of the model reflect the fact that debt's position in the financial pecking-order declines as ξ is increased. It is also worth noting that the average debt-to-assets ratio decreases while the frequency of negative debt increases with ξ. Apparently, the reduction in leverage causes the firm to react more aggressively to changes in the expected marginal product of capital. To see this, note that the variance of investment is increasing in ξ.

Table VIII shows how various constraint proxies vary with underlying structural parameters. We begin first with a discussion of the behavior of the cash flow coefficient, given the debate that centers around its interpretation. First note that the cash flow coefficient is very sensitive to the curvature of the profit function, as determined by α. This result is consistent with Gomes (2001) and Alti (2003), who find that violations of Hayashi's (1982) linear homogeneity assumption can result in a significant cash flow coefficient even if financing is frictionless. Intuitively, curvature in the profit function drives a wedge between marginal and average q. This wedge gives cash flow some predictive power in investment regressions.

[Table VIII about here]

In the simulated data, the cash flow coefficient is decreasing in each of the λ parameters. This is consistent with the static model of Kaplan and Zingales (2000). Kaplan and Zingales show that if W is defined as internal funds, profits are defined as I^{α}, $\alpha \in (0,1)$, and external equity costs are defined as $\lambda x + x^2$, then $\partial^2 I / \partial\lambda\partial W < 0$. That is, as in our model, the sensitivity of investment to internal funds is decreasing in the equity cost parameter. It should be noted that our estimated dynamic model also features concave profits and convex costs of external funds. Hence, the similarity in our conclusions is not surprising. In a closely related paper, Moyen (2004) finds in dynamic simulations that firms prohibited from issuing external equity exhibit lower cash flow coefficients than unconstrained firms. The intuition behind our findings and those in Moyen (2004) are also related. Curvature in the profit function causes cash flow to have independent predictive power in investment regressions. As Table VII shows, increases in the cost of external equity cause the firm to respond less aggressively to changes in the expected marginal product of capital. Consequently, the cash flow coefficient is decreasing in the λ parameters. Essentially, the results in Table VIII show that Moyen's argument also holds when one confines attention to local perturbations in the cost of external equity function.

As first shown by Gomes (2001), the potential failure of average q to control perfectly for shocks to the investment opportunity set is one of the problems associated with reduced-form regressions of investment on average q and cash flow. This highlights a benefit of the natural experiment approach employed by Blanchard et al. (1994), Lamont (1997), and Rauh (2004). These studies attempt to isolate surges in discretionary funds that are uncorrelated with shocks to the investment opportunity set. However, Kaplan and Zingales (1997, 2000) argue that the positive response of investment to windfalls is only sufficient to reject the null of frictionless financing. Because

Kaplan and Zingales compute a partial derivative, they hold fixed the investment opportunity set. Under concave profits and convex costs of external equity they find that, even if one can control for investment opportunities, investment cash flow sensitivity is not necessarily indicative of the magnitude of the costs of external funds. (See equation (2) in Kaplan and Zingales (2000).) We refer the interested reader to Fazzari et al. (2000) for a response.

In contrast, the cash flow coefficient increases with ξ in the simulated data. As Table VII shows, higher bankruptcy costs cause the firm to choose less debt and to hold more cash. A firm with less debt invests more, because the underinvestment incentive is less pronounced. Further, a firm sitting on a pool of cash invests even more aggressively when hit with a positive shock. Thus, endogenous reductions in leverage contribute to a higher cash flow coefficient. In addition, a firm that faces higher bankruptcy costs may attempt to reduce the probability of bankruptcy by installing more capital. Ceteris paribus, a higher capital stock reduces the probability of incurring costs of financial distress.

Consistent with theory, the sensitivity of investment to average q declines with each of the four financing cost parameters. Intuitively, financing frictions make the firm less responsive to changes in the shadow value of installed capital, which is imperfectly correlated with average q. We stress that this result should be interpreted with caution. In particular, the q coefficient may only be a reliable guide as to the cost of external funds in *simulated* data. In real-world data, measurement error in q may limit the utility of the q coefficient as a guide to the costs of external funds.

Finally, consider the behavior of the three constraint indexes. All of them decline when bankruptcy costs increase. That is, all three indexes are unreliable guides as to the magnitude of bankruptcy costs. This is because all three indexes use leverage ratios as an indicator for a firm being more constrained. If bankruptcy costs are increased, the simulated firm optimally substitutes equity for debt in its financial structure, which makes it appear less constrained. Conversely, increases in the cost of external equity cause the simulated firm to substitute debt for equity. Due to their large positive loadings on leverage, the Cleary and Whited-Wu indexes increase when costs of external equity are increased.

The more general message of this analysis is that the selection of constraint proxies depends on the task at hand. A proxy intended to gauge a firm's *need for external funds* should not be employed as a gauge of the firm's *cost of external funds*. Small firm size seems best suited as a proxy for high costs of external funds. The Cleary, Whited-Wu, and Kaplan-Zingales indexes seem best suited as proxies for a high need for external funds.

V. Conclusions

This paper uses SMM to estimate costs of external finance. We first present a dynamic structural model that endogenizes the relevant choice variables of the firm, namely, investment, cash distributions, leverage and default. We then take this model to the data. We estimate which constellation of financial frictions best explains observed financing and investment behavior, that is, minimizes the distance between model-generated moments and real-world moments. The estimated financing costs for large firms can be reconciled with low underwriting fees and low costs of bankruptcy. However, small firms appear to face large financing frictions, consistent with theories emphasizing adverse selection.

When the structural model is confronted with the real-world data, some deficiencies are revealed, suggesting natural directions for future research. The model overshoots the variance of investment somewhat, suggesting the need to incorporate frictions on the real side. More pronounced is the model's tendency to overshoot the variance of corporate cash distributions. This is consistent with Lintner's statistical model of corporate dividends. However, we still lack a theoretical rationale for dividend smoothing arising from optimizing behavior.

Appendix A: Proofs

Proof of Lemma 1: In the interest of brevity and keeping our notation consistent with that in Stokey and Lucas (SL) (1989), let

$$\begin{aligned} F(\widetilde{w}, k', b') &\equiv \Phi_d[\widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k')] - \Phi_i[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')] \\ \beta &\equiv \frac{1}{1 + r(1 - \tau_i)}. \end{aligned}$$

Partitioning the constraint correspondence as

$$\begin{aligned} \Gamma^+(z) &\equiv \{(k', b') \in \Gamma(z) : \widetilde{w} + b' - k' \geq 0\} \\ \Gamma^-(z) &\equiv \{(k', b') \in \Gamma(z) : \widetilde{w} + b' - k' \leq 0\}, \end{aligned}$$

we may express the Bellman operator (T) for this problem as follows, for arbitrary $f \in C(\Omega)$:

$$(Tf)(\widetilde{w}, z) \equiv \max \left\{ \begin{array}{ll} \max\limits_{(k',b') \in \Gamma^-(z)} & -[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')] + \beta \int_Z f[\widetilde{w}(k', b', z, z'), z'] Q(z, dz'), \\ \max\limits_{(k',b') \in \Gamma^+(z)} & \widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k') + \beta \int_Z f[\widetilde{w}(k', b', z, z'), z'] Q(z, dz') \end{array} \right\}$$

where the constraints are as specified in (12). Because Ω is compact, Weierstrass' Theorem ensures that each $f \in C(\Omega)$ is bounded.

We first claim that

$$T : C(\Omega) \to C(\Omega).$$

Fix $f \in C(\Omega)$ and consider first the problem

$$\max_{(k',b') \in \Gamma^+(z)} \quad \widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k') + \beta \int_Z f[\widetilde{w}(k', b', z, z'), z'] Q(z, dz').$$

Continuity of the function $\widetilde{r}$ implies continuity of $\widetilde{w}$. Lemma 9.5$'$ in SL implies that the expectation above is bounded and continuous. From the Theorem of the Maximum, the function

$$f^+(\widetilde{w}, z) \equiv \max_{(k',b') \in \Gamma^+(z)} \quad \widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k') + \beta \int_Z f[\widetilde{w}(k', b', z, z'), z'] Q(z, dz')$$

is continuous, and hence bounded. By the same reasoning, the function

$$f^-(\widetilde{w}, z) \equiv \max_{(k',b') \in \Gamma^-(z)} \quad -[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')] + \beta \int_Z f[\widetilde{w}(k', b', z, z'), z'] Q(z, dz')$$

is continuous, and hence bounded.

Thus, we can write the Bellman operator as

$$(Tf)(\widetilde{w},z) \equiv \max\left\{f^{+}(\widetilde{w},z), f^{-}(\widetilde{w},z)\right\},$$

which is continuous and bounded. This establishes the first claim.

We next show that T satisfies Blackwell's sufficient conditions for a contraction mapping, stated as Theorem 3.3 in SL. To establish monotonicity, consider arbitrary functions f_1 and f_2 in $C(\Omega)$, where $f_1 \leq f_2$ on Ω. For $i = 1, 2$, we can define the same partitioned maximization problems as above, with

$$(Tf_i)(\widetilde{w},z) \equiv \max\left\{f_i^{+}(\widetilde{w},z), f_i^{-}(\widetilde{w},z)\right\}.$$

Let (k'_*, b'_*) be the optimal policies corresponding to the value $f_1^{+}(\widetilde{w},z)$. It follows that

$$\begin{aligned} f_1^{+}(\widetilde{w},z) &= \widetilde{w} + b'_* - k'_* - T^d(\widetilde{w} + b'_* - k'_*) + \beta \int_Z f_1[\widetilde{w}(k'_*, b'_*, z, z'), z']Q(z, dz') \\ &\leq \widetilde{w} + b'_* - k'_* - T^d(\widetilde{w} + b'_* - k'_*) + \beta \int_Z f_2[\widetilde{w}(k'_*, b'_*, z, z'), z']Q(z, dz') \\ &\leq f_2^{+}(\widetilde{w},z). \end{aligned}$$

The first inequality follows from the hypothesis $f_1 \leq f_2$ and the second follows from a standard dominance argument. By the same reasoning

$$\begin{aligned} f_1^{-}(\widetilde{w},z) &\leq f_2^{-}(\widetilde{w},z) \\ &\Rightarrow Tf_1(\widetilde{w},z) \leq Tf_2(\widetilde{w},z). \end{aligned}$$

Now fix scalar $a \geq 0$ and $f \in C(\Omega)$. We have

$$\begin{aligned} [T(f+a)](\widetilde{w},z) &\equiv \max\left\{\begin{array}{ll} \max\limits_{(k',b') \in \Gamma^{-}(z)} & -[k' - \widetilde{w} - b' + \Lambda(k' - \widetilde{w} - b')] + \beta \int_Z [f(\widetilde{w}(k', b', z, z'), z') + a]Q(z, dz'), \\ \max\limits_{(k',b') \in \Gamma^{+}(z)} & \widetilde{w} + b' - k' - T^d(\widetilde{w} + b' - k') + \beta \int_Z [f(\widetilde{w}(k', b', z, z'), z') + a]Q(z, dz') \end{array}\right\} \\ &= \beta a + (Tf)(\widetilde{w},z). \end{aligned}$$

This establishes discounting. Hence, T is a contraction mapping.■

Proof of Proposition 3: Let $C'(\Omega)$ and $C''(\Omega)$ be the space of all functions in $C(\Omega)$, that are, respectively, weakly and strictly increasing in their first argument. SL's Corollary 1 to the Contraction Mapping Theorem shows that

$$T[C'(\Omega)] \subseteq C''(\Omega) \Rightarrow V \in C''(\Omega).$$

Fix $f \in C'(\Omega)$ and $z \in Z$. Assume the policy pairs (k_1', b_1') and (k_2', b_2') attain the supremum for the firm starting with revised net worth equal to $\widetilde{w}_1$ and $\widetilde{w}_2$, respectively, where $\widetilde{w}_1 > \widetilde{w}_2$. Then

$$\begin{aligned}
(Tf)(\widetilde{w}_1, z) &= F(\widetilde{w}_1, k_1', b_1') + \beta \int_Z f[\widetilde{w}(k_1', b_1', z, z'), z']Q(z, dz') \\
&\geq F(\widetilde{w}_1, k_2', b_2') + \beta \int_Z f[\widetilde{w}(k_2', b_2', z, z'), z']Q(z, dz') \\
&> F(\widetilde{w}_2, k_2', b_2') + \beta \int_Z f[\widetilde{w}(k_2', b_2', z, z'), z']Q(z, dz') \\
&= (Tf)(\widetilde{w}_2, z).
\end{aligned}$$

The first inequality follows from that fact that (k_1', b_1') must weakly dominate (k_2', b_2') for the firm with revised net worth $\widetilde{w}_1$, because both firms have the same feasible set $\Gamma(z)$. The $>$ sign follows from the fact that F is strictly increasing in its first argument. This establishes

$$T[C'(\Omega)] \subseteq C''(\Omega).\blacksquare$$

Proof of Proposition 4: Let $C'(\Omega)$ be the space of all functions in $C(\Omega)$ that are nondecreasing in their second argument. SL's Corollary 1 to the Contraction Mapping Theorem shows that

$$T[C'(\Omega)] \subseteq C'(\Omega) \Rightarrow V \in C'(\Omega).$$

Fix $f \in C'(\Omega)$ and $\widetilde{w}$. Assume that the policy pairs (k_1', b_1') and (k_2', b_2') attain the supremum for the firm starting with the shocks z_1 and z_2, respectively, where $z_1 > z_2$. Then

$$\begin{aligned}
(Tf)(\widetilde{w}, z_1) &= F(\widetilde{w}, k_1', b_1') + \beta \int_Z f[\widetilde{w}(k_1', b_1', z_1, z'), z']Q(z_1, dz') \\
&\geq F(\widetilde{w}, k_2', b_2') + \beta \int_Z f[\widetilde{w}(k_2', b_2', z_1, z'), z']Q(z_1, dz') \\
&\geq F(\widetilde{w}, k_2', b_2') + \beta \int_Z f[\widetilde{w}(k_2', b_2', z_2, z'), z']Q(z_2, dz') \\
&= Tf(\widetilde{w}, z_2).
\end{aligned}$$

The first inequality follows from that fact that (k_1', b_1') must weakly dominate (k_2', b_2') for the firm facing the shock z_1, because $\Gamma(z_2) \subseteq \Gamma(z_1)$ by hypothesis. The second inequality follows from the fact that F is invariant to z, $\widetilde{w}$ is nondecreasing in its third argument, and Q is monotone.$\blacksquare$

Appendix B: Simulated Method of Moments

The goal of SMM is to estimate a vector of unknown structural parameters, say θ^*, by matching a set of *simulated moments*, denoted as m^*, with corresponding *data moments*, denoted as M^*. The candidates for the moments to be matched include simple summary statistics and ordinary least squares regression coefficients.

Without loss of generality, the data moments can be represented as the solution to the maximization of a criterion function

$$\hat{M}_N = \arg\max_M J\left(Y_N, M\right),$$

where Y_N is a data matrix of length N. We first estimate $\hat{M}_N$. Then we construct S data sets based on simulations of the model under a given parameter vector, θ. For each simulated data set, we estimate m^* by maximizing an analogous criterion function

$$\hat{m}_n^s\left(\theta\right) = \arg\max_m j\left(y_n^s, m\right),$$

where y_n^s is a simulated data matrix of length n, and where, $\hat{m}_n^s\left(\theta\right)$ is expressed as an explicit function of the structural parameters utilized in that particular round of simulations. The SMM estimator of θ^* solves

$$\begin{aligned}\widehat{\theta} &= \arg\min_\theta \left[\hat{M}_N - \frac{1}{S}\sum_{s=1}^{S}\hat{m}_n^s\left(\theta\right)\right]' \hat{W}_N \left[\hat{M}_N - \frac{1}{S}\sum_{s=1}^{S}\hat{m}_n^s\left(\theta\right)\right] \\ &\equiv \arg\min_\theta \hat{G}_N' \hat{W}_N \hat{G}_N,\end{aligned}$$

where $\hat{W}_N$ is an arbitrary positive definite matrix that converges in probability to a deterministic positive definite matrix W. The optimal weighting matrix is

$$\left[N \operatorname{var}\left(\hat{M}_N\right)\right]^{-1}.$$

We use the influence-function approach in Erickson and Whited (2000) to calculate this covariance matrix. Specifically, we stack the influence functions for each of our moments and then form the covariance matrix by taking the sample average of the inner product of this stack.

The indirect estimator is asymptotically normal for fixed S. Define $j^* \equiv \operatorname{plim}_{n\to\infty}\left(j_n\right)$. Then

$$\sqrt{N}\left(\widehat{\theta} - \theta^*\right) \overset{d}{\longrightarrow} \mathcal{N}\left(0, \operatorname{avar}(\widehat{\theta})\right),$$

where

$$\text{avar}(\widehat{\theta}) \equiv \left(1+\frac{1}{S}\right)\left[\frac{\partial j^*}{\partial\theta\partial m'}\left(\frac{\partial j^*}{\partial m}\frac{\partial j^{*\prime}}{\partial m}\right)^{-1}\frac{\partial j^*}{\partial m\partial\theta'}\right]^{-1}.$$

Further, the technique provides a test of the overidentifying restrictions of the model, with

$$\frac{NS}{1+S}\hat{G}_N'\hat{W}_N\hat{G}_N$$

converging in distribution to a χ^2 with degrees of freedom equal to the dimension of M minus the dimension of θ.

We use a minimization algorithm, namely, simulated annealing, that avoids local minima. Finally, we perform a check of the numerical condition for local identification. Let $\hat{m}_n^s$ be a subvector of m with the same dimension as θ. Local identification demands that the Jacobian determinant, $\det\left(\partial\hat{m}_n^s\left(\theta\right)/\partial\theta\right)$, be nonzero. This condition can be interpreted loosely as saying that the moments (m), are informative about the structural parameters (θ). If this were not the case, not only would $\det\left(\partial\hat{m}_n^s\left(\theta\right)/\partial\theta\right)$ be near zero, but sample counterpart to the term $\partial j^*/\partial\theta\partial m'$ would be as well—a condition that would cause the parameter standard errors to blow up.

Appendix C: Data

The data are from the combined annual, research, and full coverage 2004 Standard and Poor's Compustat industrial files. The sample is selected by first deleting any firm-year observations with missing data, or for which total assets, the gross capital stock, or sales are either zero or negative. A firm is included in the sample only if it has at least two consecutive years of complete data. Finally, a firm is omitted if its primary SIC is between 4900 and 4999, between 6000 and 6999, or greater than 9000, as the model is inappropriate for regulated, financial, or public service firms. After trimming the top and bottom 2% of the variables in the data set, we end up with an unbalanced panel of firms from 1988 to 2001 with between 2349 and 2587 observations per year. Data variables are defined as follows: Book assets is Compustat Item 6; gross capital stock is Item 7; investment is the difference between Items 30 and 107; cash flow is the sum of Items 18 and 14; equity issuance is Item 108; total long-term debt is Item 9 plus Item 34; total cash distributions is the sum of Item 19, Item 21, and Item 115; the stock of cash is Item 1; and sales is Item 12. Net debt is total long-term debt less cash. Average q is calculated as in Erickson and Whited (2000).

The Kaplan-Zingales index comes from Kaplan and Zingales (1997), who examine the annual reports of the 49 firms in Fazzari et al.'s (1988) "constrained" sample, using this information to rate the firms on a financial constraints scale from one to four. They run an ordered logit of this scale on observable firm characteristics. Several authors use these logit coefficients on data from a broad sample of firms to construct a "synthetic Kaplan-Zingales index" in order to measure finance constraints. It is worth noting that Kaplan and Zingales do not claim that their coefficients could be generalized to large samples. The index is constructed as

$$-1.001909CF + 3.139193TLTD - 39.36780TDIV - 1.314759CASH + 0.2826389Q,$$

where CF is the ratio of cash flow to book assets, $TLTD$ is the ratio of total long-term debt to book assets, $TDIV$ is the ratio of total dividends to book assets, $CASH$ is the ratio of the stock of cash to book assets, and Q is the market-to-book ratio, whose numerator is defined as book assets minus book equity (item 60) minus balance sheet deferred taxes (item 7) plus the market value of equity (item 199 times item 25). The denominator is book assets.

The second index is from Whited and Wu (2005), who estimate the Lagrange multiplier on a dividend nonnegativity constraint in an investment Euler equation. The fitted multiplier is

$$-0.091CF - 0.062DIVPOS + 0.021TLTD - 0.044LNTA + 0.102ISG - 0.035SG,$$

in which $DIVPOS$ is an indicator that equals one if the firm pays dividends, and zero otherwise, SG is own-firm real sales growth, ISG is three-digit industry sales growth, and $LNTA$ is the natural log of book assets.

The third index is from Cleary (1999), who uses discriminant analysis to construct a "Z-score" for the firm's likelihood of increasing or decreasing dividend payments. The index, as implied by Table II in Cleary, is

$$-0.11905CURAT - 1.903670TLTD + 0.00138COVER + 1.45618IMARG + 2.03604SG - 0.04772SLACK,$$

in which $CURAT$ is the ratio of current assets to current liabilities, the numerator of $COVER$ is earnings before interest and taxes, the denominator of $COVER$ is (interest expense + preferred dividend payments)$/\,(1 - \tau_c)$, $IMARG$ is the ratio of net income to net sales, the numerator of $SLACK$ is cash plus short-term investments $+(0.5 \times$ inventory$)+0.7\,$(accounts receivable) $-$ short term loans, and the denominator of $SLACK$ is net fixed assets. Current assets is Item 4; current liabilities is Item 5; earnings before interest and taxes is Item 13 minus Item 14; interest expense is Item 15; preferred dividend payments is Item 9; net income is Item 18; net sales is Item 13, cash plus short term investments is Item 1; inventory is Item 3; accounts receivable is Item 2; short terms loans is Item 196; and net fixed assets is Item 8. Extreme values of these variables are winsorized as in Cleary (1999). We multiply the Cleary index by -1 so that it is increasing in the likelihood of facing costly external finance.

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Footnotes

1. The same effect surfaces in the model of Almeida et al. (2005) if one decreases the liquidation value parameter τ.

2. Cooper and Ejarque (2001) show that the assumed profit function can be derived for a price-setter with constant returns to scale subject to demand, productivity, and input price shocks.

3. Fixed costs can result in nonunique optimal financing policies. In such cases, the numerical solutions assume that the firm opts for the policy with zero external equity.

4. Although we allow the parties to renegotiate in default, the costly state verification framework assumes the lender can commit to the costly "audit" ex ante. Stochastic audit schemes dominate if there are no taxes. See Hart (1995). However, such a contract may not qualify for interest deductions because the tax courts use seniority as a litmus test for defining debt.

5. The cost of capital for debt-financed investments is $r(1 - \tau_i) + \delta$. See Stiglitz (1973). A firm would never invest beyond the point at which its marginal product of capital in the best state equals δ.

6. This effect is also present in the continuous-time model of Leland (1994), for example. See his Figure 3.

7. The third argument in the function w is actually irrelevant, because the firm is saving rather than borrowing.

8. This assumption is not utilized in the numerical analysis. In order to establish differentiability one must establish concavity. In the absence of fixed costs, Cooley and Quadrini (2001) establish concavity under restrictions on probability densities. The technical problem is that revised net worth is convex near default. A second issue is that fixed costs cause the dividend to be convex at zero.

9. The analog of condition (26) is equation (15) in Moyen (2004). In her model, there is an additional term because bankruptcy costs are assumed to be proportional to the face value

of debt. Our bankruptcy costs are proportional to the real capital stock.

10. Michaelides and Ng (2000) find that good finite-sample performance of an indirect inference estimator requires a simulated sample that is approximately ten times as large as the actual data sample.

11. We thank Michael Roberts for suggesting the inclusion of covariances between investment and external financing. Simulations indicate that such moments are indeed informative.

12. One final issue is unobserved heterogeneity in the data from Compustat. The simulations produce independent and identically distributed firms. To remove the effects of heterogeneity from the autoregression, we include time dummies and remove firm fixed effects by differencing the data.

13. Our estimation results are reasonably robust to changes in the parameters we do not estimate. For example, if we set the depreciation rate equal to 10%, the financing parameters decrease on average by only 14%. Similarly, if we decrease the corporate income tax rate to 35%, estimated deadweight bankruptcy costs decrease by 30%, but the rest of the estimates are affected little.

14. The existence of a bond rating has also been used widely as an indicator of finance constraints. However, in our data set only 19% of the firms have bond ratings, so we are unable to obtain precise parameter estimates.

Table I

Simulated Moments Estimation for the Full Sample

Calculations are based on a sample of nonfinancial, unregulated firms from the annual 2004 COMPUSTAT industrial files. The sample period is 1988 to 2001. Estimation is done with SMM, which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from the data. The simulated panel of firms is generated from the model in Section II, and contains 20,000 firms over 200 time periods, where only the last 14 time periods are kept for each firm. The first panel reports the simulated and estimated moments. The second panel reports the estimated structural parameters, with standard errors in parentheses. λ_0, λ_1, and λ_2 are the fixed, linear, and quadratic costs of equity issuance. ϕ governs the shape of the distributions tax schedule, with a lower value for ϕ corresponding to a lower marginal tax rate. ξ is the bankruptcy cost parameter, with total bankruptcy costs equal to ξ times the capital stock. σ_ε is the standard deviation of the innovation to $\ln(z)$, in which z is the shock to the revenue function. ρ is the serial correlation of $\ln(z)$. χ^2 is a chi-squared statistic for the test of the overidentifying restrictions. In parentheses is its p-value.

Panel A: Moments		
	Actual Moments	Simulated Moments
Average Equity Issuance/Assets	0.0892	0.0963
Variance of Equity Issuance/Assets	0.0911	0.0847
Variance of Investment/Assets	0.0068	0.0117
Frequency of Equity Issuance	0.1751	0.2305
Payout Ratio	0.2226	0.2026
Frequency of Negative Debt	0.3189	0.3258
Variance of Distributions	0.0013	0.0037
Average Debt-Assets Ratio (Net of Cash)	0.1204	0.1104
Covariance of Investment and Equity Issuance	0.0004	0.0005
Covariance of Investment and Leverage	-0.0018	-0.0025
Serial Correlation of Income/Assets	0.5121	0.5661
Standard Deviation of the Shock to Income/Assets	0.1185	0.1057

Panel B: Parameter Estimates

α	λ_0	λ_1	λ_2	ξ	ϕ	σ_ε	ρ	χ^2
0.627	0.598	0.091	0.0004	0.104	0.732	0.118	0.684	8.018
(0.219)	(0.233)	(0.026)	(0.0008)	(0.059)	(0.844)	(0.042)	(0.349)	(0.091)

Table II

Simulated Moments Estimation for Small Firms

Calculations are based on a sample of nonfinancial, unregulated firms from the annual 2004 COMPUSTAT industrial files, in which only firms in the lower third of the distribution of book assets are retained. The sample period is 1988 to 2001. Estimation is done with SMM, which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from the data. The simulated panel of firms is generated from the model in Section II, and contains 20,000 firms over 200 time periods, where only the last 14 time periods are kept for each firm. The first panel reports the simulated and estimated moments. The second panel reports the estimated structural parameters, with standard errors in parentheses. λ_0, λ_1, and λ_2 are the fixed, linear, and quadratic costs of equity issuance. ϕ governs the shape of the distributions tax schedule, with a lower value for ϕ corresponding to a lower marginal tax rate. ξ is the bankruptcy cost parameter, with total bankruptcy costs equal to ξ times the capital stock. σ_ε is the standard deviation of the innovation to $\ln(z)$, in which z is the shock to the revenue function. ρ is the serial correlation of $\ln(z)$. χ^2 is a chi-squared statistic for the test of the overidentifying restrictions. In parentheses is its p-value.

Panel A: Moments

	Actual Moments	Simulated Moments
Average Equity Issuance/Assets	0.1243	0.1389
Variance of Equity Issuance/Assets	0.1400	0.1352
Variance of Investment/Assets	0.0081	0.0171
Frequency of Equity Issuance	0.2391	0.3307
Payout Ratio	0.1252	0.0971
Frequency of Negative Debt	0.4284	0.3990
Variance of Distributions	0.0031	0.0052
Average Debt-Assets Ratio (Net of Cash)	0.0840	0.0614
Covariance of Investment and Equity Issuance	0.0007	0.0010
Covariance of Investment and Leverage	-0.0011	-0.0019
Serial Correlation of Income/Assets	0.4083	0.4123
Standard Deviation of the Shock to Income/Assets	0.1662	0.1419

Panel B: Parameter Estimates

α	λ_0	λ_1	λ_2	ξ	ϕ	σ_ε	ρ	χ^2
0.693	0.951	0.120	0.0004	0.151	0.831	0.159	0.498	9.182
(0.302)	(0.495)	(0.039)	(0.0010)	(0.072)	(0.711)	(0.081)	(0.280)	(0.057)

Table III

Simulated Moments Estimation for Large Firms

Calculations are based on a sample of nonfinancial, unregulated firms from the annual 2004 COMPUSTAT industrial files, in which only firms in the upper third of the distribution of book assets are retained. The sample period is 1988 to 2001. Estimation is done with SMM, which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from the data. The simulated panel of firms is generated from the model in Section II, and contains 20,000 firms over 200 time periods, where only the last 14 time periods are kept for each firm. The first panel reports the simulated and estimated moments. The second panel reports the estimated structural parameters, with standard errors in parentheses. λ_0, λ_1, and λ_2 are the fixed, linear, and quadratic costs of equity issuance. ϕ governs the shape of the distributions tax schedule, with a lower value for ϕ corresponding to a lower marginal tax rate. ξ is the bankruptcy cost parameter, with total bankruptcy costs equal to ξ times the capital stock. σ_ε is the standard deviation of the innovation to $\ln(z)$, in which z is the shock to the revenue function. ρ is the serial correlation of $\ln(z)$. χ^2 is a chi-squared statistic for the test of the overidentifying restrictions. In parentheses is its p-value.

Panel A: Moments

	Actual Moments	Simulated Moments
Average Equity Issuance/Assets	0.0662	0.0765
Variance of Equity Issuance/Assets	0.0507	0.0784
Variance of Investment/Assets	0.0052	0.0126
Frequency of Equity Issuance	0.1470	0.1905
Payout Ratio	0.3212	0.2852
Frequency of Negative Debt	0.2282	0.2402
Variance of Distributions	0.0009	0.0013
Average Debt-Assets Ratio (Net of Cash)	0.1452	0.1610
Covariance of Investment and Equity Issuance	0.0004	0.0009
Covariance of Investment and Leverage	-0.0015	-0.0031
Serial Correlation of Income/Assets	0.6455	0.6340
Standard Deviation of the Shock to Income/Assets	0.0796	0.0735

Panel B: Parameter Estimates

α	λ_0	λ_1	λ_2	ξ	ϕ	σ_ε	ρ	χ^2
0.577	0.389	0.053	0.0002	0.084	0.695	0.086	0.791	7.145
(0.235)	(0.302)	(0.022)	(0.0003)	(0.055)	(0.920)	(0.045)	(0.322)	(0.128)

Table IV

Summary Statistics from Simulated Constrained and Unconstrained Firms

This table presents summary statistics from four simulated firms. The large firms are parameterized according to the estimates in Table III, and the small firms are parameterized according to the estimates in Table II. The unconstrained firms have all financing parameters set to zero.

	Investment/ Assets	Tobin's q	Net Debt/ Assets	Equity Issuance/ Assets	Frequency of Equity Issuance
Large Firms					
Constrained	0.140	1.592	0.160	0.077	0.191
Unconstrained	0.151	1.913	0.178	0.081	0.203
Small Firms					
Constrained	0.125	2.913	0.061	0.139	0.331
Unconstrained	0.159	3.724	0.099	0.142	0.386

Table V

Estimates of the Cost of External Funds: Debt Issuance Costs

Calculations are based on a sample of nonfinancial, unregulated firms from the annual 2004 COMPUSTAT industrial files. The sample period is 1988 to 2001. Estimation is done with SMM, which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from the data. The simulated panel of firms is generated from the model in Section II, and contains 20,000 firms over 200 time periods, where only the last 14 time periods are kept for each firm. The table reports the estimated structural parameters, with standard errors in parentheses. The KZ, WW, and Cleary indexes are indicators of the severity of finance constraints from Kaplan and Zingales, Whited and Wu, and Cleary, respectively. The KZ and WW indexes are increasing in the degree of financial constraints, whereas the Cleary index is decreasing. We multiply the Cleary index by -1 to make it comparable to the other indices. λ_0, λ_1, and λ_2 are the fixed, linear, and quadratic costs of equity issuance. ϕ governs the shape of the distributions tax schedule, with a lower value for ϕ corresponding to a lower marginal tax rate. ξ is the bankruptcy cost parameter, with total bankruptcy costs equal to ξ times the capital stock. σ_ε is the standard deviation of the innovation to $\ln(z)$, in which z is the shock to the revenue function. ρ is the serial correlation of $\ln(z)$. χ^2 is a chi-squared statistic for the test of the overidentifying restrictions. In parentheses is its p-value.

	α	λ_0	λ_1	λ_2	ξ	ϕ	σ_ε	ρ	χ^2
Full Sample									
	0.643	0.601	0.095	0.0004	0.109	0.780	0.122	0.669	7.665
	(0.288)	(0.259)	(0.036)	(0.0003)	(0.063)	(0.752)	(0.040)	(0.351)	(0.105)
Large Firms									
	0.589	0.411	0.061	0.0002	0.073	0.837	0.091	0.732	7.146
	(0.249)	(0.283)	(0.020)	(0.0006)	(0.058)	(1.003)	(0.051)	(0.328)	(0.282)
Small Firms									
	0.701	1.010	0.129	0.0005	0.141	0.702	0.152	0.511	9.343
	(0.327)	(0.481)	(0.037)	(0.0009)	(0.066)	(0.824)	(0.073)	(0.271)	(0.025)

Table VI

Estimates of the Cost of External Funds: Alternative Sample Splits

Calculations are based on a sample of nonfinancial, unregulated firms from the annual 2004 COMPUSTAT industrial files. The sample period is 1988 to 2001. Estimation is done with SMM, which chooses structural model parameters by matching the moments from a simulated panel of firms to the corresponding moments from the data. The simulated panel of firms is generated from the model in Section II, and contains 20,000 firms over 200 time periods, where only the last 14 time periods are kept for each firm. The table reports the estimated structural parameters, with standard errors in parentheses. λ_0, λ_1, and λ_2 are the fixed, linear, and quadratic costs of equity issuance. ϕ governs the shape of the distributions tax schedule, with a lower value for ϕ corresponding to a lower marginal tax rate. ξ is the bankruptcy cost parameter, with total bankruptcy costs equal to ξ times the capital stock. σ_ε is the standard deviation of the innovation to $\ln(z)$, in which z is the shock to the revenue function. ρ is the serial correlation of $\ln(z)$. χ^2 is a chi-squared statistic for the test of the overidentifying restrictions. In parentheses is its p-value.

	α	λ_0	λ_1	λ_2	ξ	ϕ	σ_ε	ρ	χ^2
Low Dividends									
	0.671	0.624	0.097	0.0004	0.117	0.821	0.135	0.590	8.599
	(0.303)	(0.362)	(0.025)	(0.0003)	(0.062)	(0.680)	(0.051)	(0.298)	(0.072)
High Dividends									
	0.588	0.484	0.074	0.0003	0.089	0.683	0.104	0.721	6.548
	(0.234)	(0.374)	(0.039)	(0.0007)	(0.066)	(0.754)	(0.037)	(0.366)	(0.162)
High WW Index									
	0.688	1.002	0.123	0.0005	0.140	0.990	0.138	0.483	8.322
	(0.327)	(0.481)	(0.037)	(0.0003)	(0.050)	(0.951)	(0.033)	(0.267)	(0.080)
Low WW Index									
	0.563	0.494	0.062	0.0003	0.083	0.633	0.097	0.704	6.889
	(0.200)	(0.189)	(0.018)	(0.0010)	(0.063)	(0.864)	(0.039)	(0.347)	(0.142)
High Cleary Index									
	0.702	0.808	0.108	0.0004	0.133	0.769	0.124	0.605	10.147
	(0.395)	(0.367)	(0.045)	(0.0008)	(0.042)	(0.882)	(0.052)	(0.482)	(0.038)
Low Cleary Index									
	0.644	0.466	0.063	0.0003	0.072	0.571	0.102	0.532	6.504
	(0.362)	(0.301)	(0.032)	(0.0007)	(0.064)	(0.914)	(0.054)	(0.269)	(0.164)
High KZ Index									
	0.611	0.502	0.082	0.0003	0.110	0.887	0.127	0.655	6.941
	(0.377)	(0.270)	(0.038)	(0.0006)	(0.055)	(1.079)	(0.056)	(0.307)	(0.139)
Low KZ Index									
	0.603	0.655	0.107	0.0004	0.073	0.416	0.103	0.692	7.628
	(0.280)	(0.359)	(0.039)	(0.0007)	(0.059)	(0.652)	(0.049)	(0.315)	(0.106)

Table VII
Sensitivity of Model Moments to Parameters

This table presents elasticities of model moments with respect to the model parameters. The baseline parameters are given in Table I. Each elasticity is calculated by simulating the model twice, once with a value of the parameter of interest 50% below its baseline value, and once with a value 50% above its baseline value. Then the change in the moment is calculated as the difference between the results from the two simulations. This difference is then divided by the change in the underlying structural parameter between the two simulations. The result is then multiplied by the ratio of the baseline structural parameter to the baseline moment.

	Baseline Moments	α	λ_0	λ_1	λ_2	ξ	ϕ	σ_e	ρ
Average Equity Issuance/Assets	0.0892	0.3976	-1.1545	-0.7550	-0.1010	1.3606	0.0214	1.7625	-0.9539
Variance of Equity Issuance/Assets	0.0911	0.6034	-0.6356	-0.6303	-0.0040	0.5033	0.0262	1.3983	-0.6883
Variance of Investment/Assets	0.0068	1.0371	-0.4725	-0.1962	-0.4607	0.4079	-0.4539	1.3979	0.5361
Frequency of Equity Issuance	0.1751	0.7305	-0.8172	-0.7319	-0.1561	1.3937	0.1256	1.0418	-0.4435
Payout Ratio	0.2226	0.0334	0.0407	0.0170	0.0184	-0.1778	-0.8514	-0.6841	0.2986
Frequency of Negative Debt	0.3189	0.3408	-0.2098	-0.2567	0.0900	0.5328	0.5967	0.7004	0.2139
Variance of Distributions	0.0013	0.4529	-0.0761	0.1819	0.1975	0.0069	-0.5561	1.6851	0.2931
Average Net Debt-Assets Ratio	0.1204	-0.3109	0.5373	0.1052	-0.0235	-1.2708	-0.6710	-0.8551	0.2538
Covariance of Investment and Equity Issuance	0.0004	0.4821	-0.3826	-0.8728	-0.0025	0.8456	-0.0674	0.8571	-0.0561
Covariance of Investment and Leverage	-0.0018	0.5002	0.8213	0.4954	0.1626	-1.6183	-0.7967	0.5636	0.1923
Serial Correlation of Income/Assets	0.5121	-0.1065	-0.0913	0.0696	0.0713	0.6314	0.0870	-0.0598	1.5608
Standard Deviation of the Shock to Incomes/Assets	0.1185	-0.6122	-0.0686	0.0553	0.0184	0.8591	0.0822	1.0443	-0.5471

Table VIII
Sensitivity of Finance Constraint Indicators to Parameters

This table presents elasticities of several popular indicators of financial constraints with respect to the model parameters. Investment-q and investment-cash flow sensitivities are the slope coefficients from a regression of the ratio of investment to capital on Tobin's q and the ratio of cash flow to capital. The KZ, WW, and Cleary indexes are indicators of the severity of finance constraints from Kaplan and Zingales, Whited and Wu, and Cleary respectively. The KZ and WW indexes are increasing in the degree of financial constraints, whereas the Cleary index is decreasing. We multiply the Cleary index by -1 to make it comparable to the other indices. The baseline parameters are given in Table I. Each elasticity is calculated by simulating the model twice, once with a value of the parameter of interest 50% below its baseline value, and once with a value 50% above its baseline value. Then the change in the moment is calculated as the difference between the results from the two simulations. This difference is then divided by the change in the underlying structural parameter between the two simulations. The result is then multiplied by the ratio of the baseline structural parameter to the baseline moment.

	Baseline Moments	α	λ_0	λ_1	λ_2	ξ	ϕ	σ_e	ρ
Investment-q Sensitivity	0.0061	0.0475	-0.3782	-0.4214	-0.1757	-0.0187	0.1428	0.4009	0.3967
Investment-Cash Flow Sensitivity	0.1917	-1.0023	-0.5493	-0.5089	-0.1577	0.5584	0.0016	0.1174	0.5093
WW Index	0.6741	0.0108	0.2890	0.2785	0.0304	-0.0390	-0.2482	-0.1746	-0.2884
Cleary Index	-0.2422	-0.1772	0.8579	0.7516	0.0211	-0.2821	-0.2499	-0.6485	0.3338
KZ Index	0.9010	0.5685	-0.0150	-0.5388	-0.0899	-0.3464	-0.5241	-0.3504	0.4543

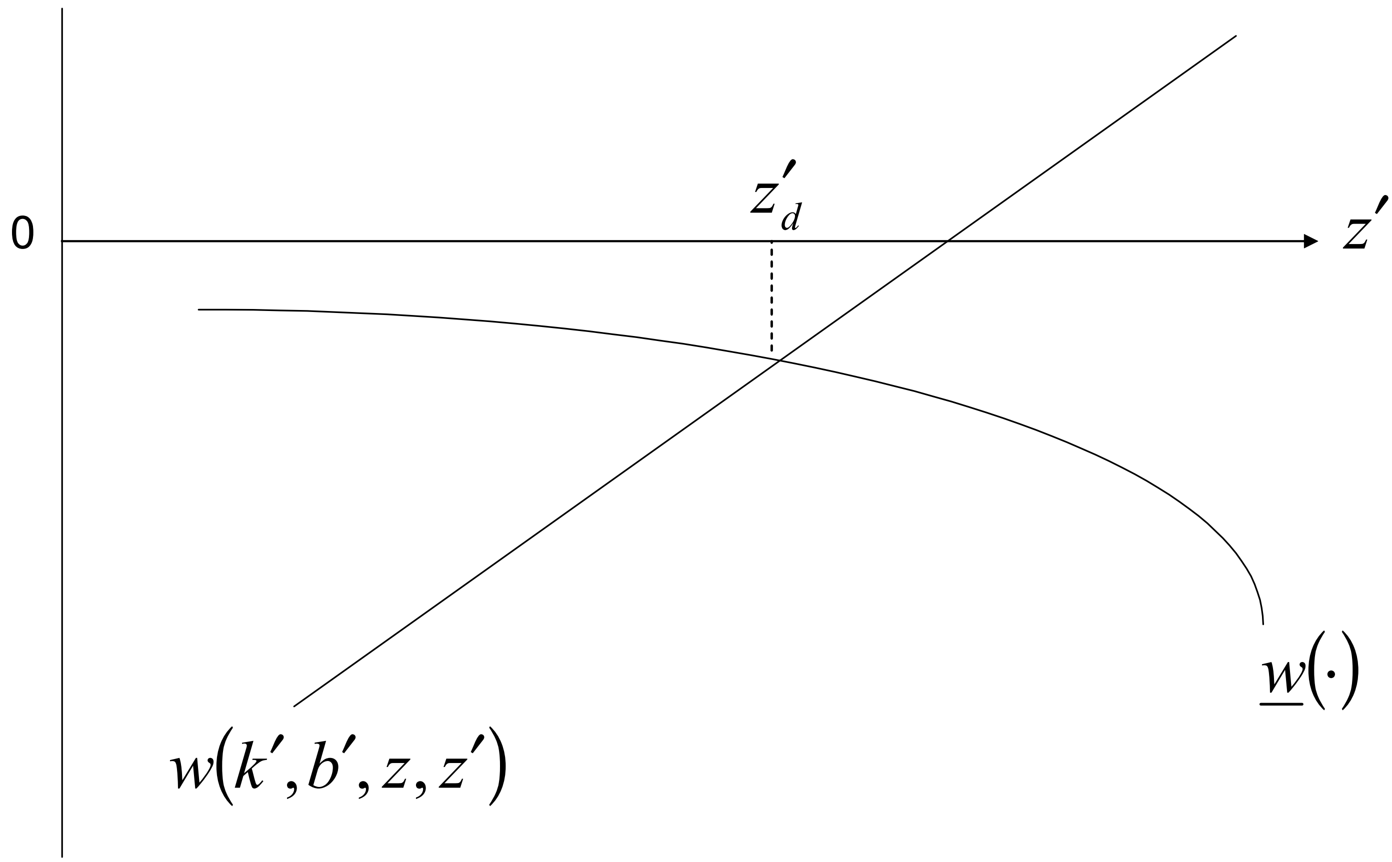

Figure 1. Endogenous default. This figure depicts the determination of the critical shock, z'_d, at which the firm chooses to default. On the horizontal axis is the shock, z'. $w(k', b', z, z')$ is the firm's realized net worth, and $\underline{w}(\cdot)$ is the firm's default schedule.