ABSTRACT

Why do corporations accumulate liquid assets? We show theoretically that intertemporal trade-offs between interest income taxation and the cost of external finance determine optimal savings. Intriguingly, we find that, controlling for Tobin’s q, saving and cash flow are negatively related because firms lower cash reserves to invest after receiving positive cash-flow shocks, and vice versa. Consistent with theory, we estimate negative propensities to save out of cash flow. We also find that income uncertainty affects saving more than do external finance constraints. Therefore, contrary to previous evidence, saving propensities reflect too many forces to be used to measure external finance constraints.

* Riddick is from the American University and Whited is from the University of Wisconsin, Madison. We are grateful for helpful comments and suggestions from an anonymous referee, an associate editor, the editors (Cam Harvey and John Graham), as well as Viral Acharya, Morris Davis, Mark Garmaise, Robert Hauswald, Chris Hennessey, Holger Mueller, Emi Nakamura, Michael Roberts, Josef Zechner, and seminar participants at the American University, Boston College, Columbia, Cornell, Duke, Université de Lausanne, École des Hautes Études Commerciales de Paris, London Business School, the London School of Economics, Northwestern, the Norwegian School of Management, Rice, Rochester, the University of Vienna, the University of Wisconsin, and Yale. Bobby Hart of Thompson Financial was helpful in data acquisition, and Alex Boquist and Michael R. Sullivan provided research assistance. Leigh Riddick also wishes to acknowledge research support from the Kogod School of Business and American University.
Why do corporations save? In other words, why do they funnel their cash flow into liquid asset holdings rather than into physical capital or into shareholder distributions? This question is challenging because such financial decisions cannot be understood in isolation of the real decisions a corporation makes. Not only is the question challenging, but it is also economically interesting in light of the tendency in recent years of both U.S. and European firms to accumulate unusually high levels of cash. More generally, savings policies matter for all firms because managers must evaluate the trade-offs between using internal and external funds to finance current and future investment. Indeed, several recent studies—for example, Almeida, Campello and Weisbach (2004) and Khurana, Martin, and Pereira (2006)—have used firms’ saving behavior to gauge the cost of external finance.

The goal of this paper is therefore twofold. Although we do not tackle directly the issue of the high level of corporate cash holdings, we shed light on this phenomenon by delving into the economics of the process whereby firms accumulate this cash. We also seek to determine whether and when corporate saving behavior can be a useful indicator of the extent to which firms face external finance constraints. We examine these issues both empirically and theoretically, and we focus on two specific determinants of saving: income uncertainty and the cost of external finance.

On the theoretical side our model provides several insights. In our infinite-horizon framework firms invest, save, produce, raise external finance, and make distributions in the face of uncertainty, physical adjustment costs, taxation, and costly equity issuance. Because interest on cash balances is taxed, the firm faces a dynamic trade-off between this tax penalty and the reduction in expected future financing costs conferred by holding cash. Therefore, the firm’s optimal saving policy depends not only on the cost of external finance, but also on the firm’s expected future financing needs, which, in turn, depend on the firm’s technology and especially on the uncertainty it faces.

In this setting we find that firms hold higher precautionary cash balances when external finance is costly or income uncertainty is high. Firms also hold more cash if their optimal investment policy is lumpy because large investments typically entail costly financing. This connection between cash holdings and large investments is consistent with the observed high corporate cash levels that have coincided with the recent wave of mergers and acquisitions, which are a form of lumpy investment.
Our most interesting predictions concern saving, that is, the change in cash. In particular, we find that, controlling for Tobin’s q, saving and cash flow are negatively correlated. Although this result of a negative sensitivity is, at first, surprising, the intuition is both economically interesting and straightforward. For example, if the firm faces positively serially correlated productivity shocks, conditional on a high shock, the firm’s cash flow rises, its capital becomes more productive, and this productivity reverts to its mean slowly. The firm therefore shifts some of its financial asset holdings into physical capital; that is, it invests and dissaves. The amount of this dissaving is typically less than the firm’s cash flow. Conversely, the firm accumulates more liquid assets in times when capital productivity is low. This substitution between physical and financial assets manifests itself in a negative sensitivity of saving to cash flow. It is important to control for Tobin’s q because, roughly speaking, this variable capitalizes the value to the firm of holding cash. Indeed, in our model the simple correlation between saving and cash flow, although small, is positive because firms often do use their cash flow to invest in financial assets.

Naturally, the negative sensitivity of saving to cash flow increases in absolute value with the serial correlation of productivity shocks. It also falls in absolute value as the shocks become more variable because the firm does not react strongly to the small amount of information in high variance shocks. Finally, the sensitivity rises in absolute value with the cost of external finance because the firm’s optimal level of cash increases with the cost of external finance. In comparison with a low cost firm, a high cost firm therefore has more slack with which to respond to profit shocks, and it saves or dissaves more aggressively to counteract part of the effects of these shocks.

This last result is particularly interesting because it indicates that although the levels and changes in cash for a firm are clearly related, a high cash level does not necessarily imply a high, positive sensitivity of the change in cash (saving) to cash flow; nor does a low cash level imply a low sensitivity. This distinction between levels and sensitivities would be impossible to uncover, for example, in a model with a one-period saving decision because in such a setting the change in cash is indistinguishable from the level. A dynamic model such as ours is therefore essential to understanding the trade-offs that affect corporate saving.

Our empirical work is closely tied to our model. To generate exact testable predictions, we solve
the model numerically and use the solution to generate a panel of simulated data. We then use these data to run a linear regression, from Almeida, Campello, and Weisbach (2004), of the change in cash levels (saving) on Tobin’s $q$ and cash flow. The coefficient on cash flow measures the saving sensitivity, which we also dub the propensity to save.

We then run the same regression on real data, primarily from the United States, but also from Canada, France, Germany, Japan, and the United Kingdom. Although we find positive and significant OLS coefficients on cash flow in all six countries, these coefficients become negative and significant when we correct econometrically for the substantial measurement error in Tobin’s $q$ documented in Erickson and Whited (2000, 2006) and Whited (2001). The difference between the two sets of results makes sense given the severe coefficient bias that measurement error can cause. As predicted by our model, we find that saving propensities are less negative in samples of firms with high income variability and more negative in samples with high serial correlation of income. However, we reject our model’s prediction that firms typically categorized as financially constrained have more negative saving propensities than their unconstrained counterparts. This result occurs because constrained firms also have highly variable income shocks, which reduce their saving propensities. In sum, from our model and empirical evidence we conclude that the variability and autocorrelation of income shocks are at least as important as the cost of external finance in determining corporate saving. Accordingly, although the sensitivity of saving to cash flow contains information about external finance constraints, too many factors influence this one correlation for it to be used as a summary measure of the cost of external finance.

Our paper fits into both the theoretical and empirical literatures on corporate saving. The theoretical model in this paper is most closely related to that in Whited (2006), who studies a firm that invests and saves in the face of costly external equity finance and fixed costs of capital adjustment. We extend the model by including a corporate income tax and convex adjustment costs, and we examine empirically the model’s implications for saving rather than for investment, as in Whited (2006). Our model is also closely related to the one in Eisfeldt and Rampini (2006), who characterize the business-cycle properties of aggregate liquidity. They calibrate a general-equilibrium model with a rich specification of uncertainty. Although many of the same economic
mechanisms at work in their model also operate in ours, the focus of the two papers is quite different in that we are interested in directly testing the implications of the model at the firm level, instead of calibration at the aggregate level. Another closely related theoretical paper is Gamba and Triantis (2006). Their model is quite general, allowing for cash holding as well as separate debt and equity finance, although, unlike us, they omit physical adjustment costs. Their main contribution is an explanation of how debt flotation costs can lead to simultaneous cash and debt holdings.

Our paper is most closely related to Almeida, Campello, and Weisbach (2004). We extend their work along both theoretical and empirical dimensions. Theoretically, their model predicts a positive propensity to save—a result that occurs for two reasons. First, in their model an increase in cash flow is not accompanied by higher capital productivity. Therefore, the firm has no incentive to transform liquid assets into physical assets, as in our model, and increased cash flow produces a pure positive income effect on saving, whereby part of the additional cash flow is used to elevate both the current and future capital stock. Second, physical assets depreciate completely between periods and cannot be used to transfer resources over time. This model feature increases the value of cash holding relative to a model in which capital depreciates more slowly. Our multiperiod model, in contrast, allows for a variety of depreciation rates and for cash flow shocks that may or may not be tied to productivity. Interestingly, our model almost always predicts negative saving propensities.

Empirically, Almeida, Campello, and Weisbach (2004) find a positive sensitivity in the data and we primarily find a negative sensitivity. The difference lies in our correction for measurement error in Tobin’s $q$. This result is puzzling in light of the argument in Almeida, Campello, and Weisbach (2004) that using the sensitivity of cash saving (instead of physical investment) to cash flow as a measure of financing constraints is immune to the measurement error issue. They explain correctly that under the null hypothesis of no financing frictions, saving should not depend on either cash flow or Tobin’s $q$. Therefore, saving can only be sensitive to cash flow in the presence of financial frictions. We elaborate on this argument by using measurement-error consistent estimators to examine the sign and magnitude of the cash-flow effect. As explained in detail below, and also as noted in Greene (1997, p. 440), measurement error in Tobin’s $q$ can affect the cash flow coefficient because
measurement error in one regressor affects all of the coefficients in a regression if the regressors are correlated with one another. In this case they are correlated because the information about future investment opportunities contained in cash flow leads naturally to a positive correlation between Tobin’s q and cash flow. Putting saving instead of investment on the left side of the regression does not eliminate this problem.

Our empirical work is related not only to Almeida, Campello, and Weisbach (2004), but also to Khurana, Martin, and Pereira (2006), who replicate the results in Almeida, Campello, and Weisbach (2004) on data from several countries; Acharya, Almeida, and Campello (2007), who examine both the propensity to save out of cash flow and the propensity to issue debt; and Sufi (2006), who uses saving propensities as an explicit metric for gauging the severity of the cost of external finance.1

The paper is organized as follows. Section I presents the model. Section II describes the model simulation and its results. Section III describes the data, Section IV presents the estimation procedure and results, and Section V concludes. A supplementary Appendix2 contains simulations that assess the estimators’ finite-sample performance and supplementary tables.

I. A Model of Cash Holding

To motivate our empirical work, we consider a discrete-time, infinite-horizon, partial-equilibrium model of investment and saving. First we describe technology, financing, and taxation. Then we move on to a description of optimal financing policies.

A. Technology and Financing

A risk-neutral firm uses capital, k, and variable factors of production, l, to produce output, and it faces a combination demand and productivity shock, z. Because the variable factors are costlessly adjustable, the firm’s per period profit function is given by \( \pi (k, z) \), where the variable factors have already been maximized out of the problem. The profit function \( \pi (k, z) \) is continuous, with \( \pi (0, z) = 0 \), \( \pi_z (k, z) > 0 \), \( \pi_k (k, z) > 0 \), \( \pi_{kk} (k, z) < 0 \), and \( \lim_{k \to \infty} \pi_k (k, z) = 0 \). Concavity of \( \pi (k, z) \) results from decreasing returns in production, a downward sloping demand curve, or both. The shock z is observed by the producer before he makes his current period decisions. It takes
values in $[\bar{z}, \bar{z}]$ and follows a first-order Markov process with transition probability $g(z', z)$, where a prime indicates a variable in the next period; $g(z', z)$ has the Feller property.

Without loss of generality, $k$ lies in a compact set. As in Gomes (2001), define $\bar{k}$ as

$$
(1 - \tau_c) \pi(\bar{k}, z) - d\bar{k} \equiv 0,
$$

where $d$ is the capital depreciation rate, $0 < d < 1$, and $\tau_c$ is the corporate income tax rate. Concavity of $\pi(k, z)$ and $\lim_{k \to \infty} \pi_k(k, z) = 0$ ensure that $\bar{k}$ is well defined. Because $k > \bar{k}$ is not economically profitable, $k$ lies in the interval $[0, \bar{k}]$. Compactness of the state space and continuity of $\pi(k, z)$ ensure that $\pi(k, z)$ is bounded.

Investment, $I$, is defined as

$$
I \equiv k' - (1 - d)k.
$$

The firm purchases and sells capital at a price of one and incurs adjustment costs that are given by

$$
A(k, k') = ck\Phi_i + a \left( \frac{k' - (1 - d)}{k} \right)^2 k.
$$

The functional form of (3) is standard in the empirical investment literature, and it encompasses both fixed and smooth adjustment costs. See, for example, Cooper and Haltiwanger (2006). The first term captures the fixed component, $ck\Phi_i$, where $c$ is a constant and $\Phi_i$ equals one if investment is nonzero and zero otherwise. The fixed cost is proportional to the capital stock so that the firm has no incentive to grow out of the fixed cost.\(^3\) The smooth component is captured by the second term, in which $a$ is a constant. Although curvature of the profit function acts to smooth investment over time in the same way that the quadratic component of (3) does, we include the quadratic component to isolate the effects of smooth adjustment costs. In contrast, curvature of the profit function not only affects investment smoothing but also the relation between firm value and profit.

We now discuss financing. The firm can hold cash, $p$, via a riskless one-period discount bond that earns taxable interest at a rate $r(1 - \tau_c)$. For simplicity, we do not model personal interest and dividend taxes. To ensure bounded savings, our model requires some penalty for holding cash. We have chosen to model a tax penalty. Other choices include agency costs, as in Eisfeldt and Rampini (2006), or a stochastic probability of default, as in Carlstrom and Fuerst (1997). To make
the choice set compact, we assume an arbitrarily high upper bound on liquid assets, $\bar{p}$. This upper bound is imposed without loss of generality because of our taxation assumptions.

All external finance takes the form of equity. This simplification allows us to highlight the interaction between technology, finance constraints, and cash holdings. Also, having a single source of external finance does not affect the qualitative outcome of the model simulations, which only rely on a difference in the costs of external and internal funds. Differentiation among the types of external financing, although important for the study of capital structure, is not required. To preserve tractability, we do not model costs of external equity as the outcome of an asymmetric information problem. Instead, we capture adverse selection costs and underwriting fees in a reduced-form fashion. Accordingly, we define gross equity issuance/distributions as

$$
e (k, k', p, p', z) \equiv (1 - \tau_c) \pi(k, z) + p - \frac{p'}{(1 + r(1 - \tau_c))} - (k' - (1 - d)k) - A (k, k'). \quad (4)$$

If $e(k, k', p, p', z) > 0$, the firm is making distributions to shareholders, and if $e(k, k', p, p', z) < 0$, the firm is issuing equity. The external equity cost function is linear-quadratic and weakly convex:

$$
\phi(e(k, k', p, p', z)) \equiv \Phi_e \left( -\lambda_0 + \lambda_1 e(k, k', p, p', z) - \frac{1}{2} \lambda_2 e(k, k', p, p', z)^2 \right) \\
\lambda_i \geq 0, \quad i = 0, 1, 2,
$$

where $\Phi_e$ equals one if $e(k, p, k', p', z) < 0$, and zero otherwise. Convexity of $\phi(e(k, p, k', p', z))$ is consistent with the evidence on underwriting fees in Altinkilic and Hansen (2000).

The firm chooses $(k', p')$ each period to maximize the value of expected future cash flows, discounting at the opportunity cost of funds, $r$. The Bellman equation for the problem is

$$
V(k, p, z) = \max_{k', p'} \left\{ e(k, k', p, p', z) + \phi(e(k, k', p, p', z)) + \frac{1}{1 + r} \int V(k', p', z') \, dg(z', z) \right\}. \quad (5)
$$

The first two terms represent the excess of cash inflows over cash outflows and the third term represents the continuation value of the firm. The model satisfies the conditions for Theorem 9.6 in Stokey and Lucas (1989), which guarantees a solution for (5). Theorem 9.8 in Stokey and Lucas (1989) ensures a unique optimal policy function, $\{k', p'\} = h(k, p, z)$, if $e(k, k', p, p', z) + \phi(e(k, k', p, p', z))$ is weakly concave in its first and third arguments. This requirement puts easily verified restrictions on $\phi(\cdot)$ that are satisfied by the functional forms chosen below.
B. Optimal Financial Policies

This subsection develops the intuition behind the model by examining its optimality conditions. To simplify the exposition of optimal policies, we assume in this subsection that $V$ is concave and once differentiable. These assumptions are not necessary for the existence of a solution to (5) or of an optimal policy function. We present optimal financial policies, heuristically, in two steps. First, we determine optimal financing under the assumption that the manager ignores the fixed costs of external equity; that is, he treats $\lambda_0 = 0$. Second, we determine whether the intramarginal benefits of equity issuance justify the fixed cost.

The optimal interior financial policy, obtained by solving the optimization problem (5), satisfies

$$1 + (\lambda_1 - \lambda_2 e) \Phi_e = \frac{1 + r (1 - \tau_c)}{1 + r} \int V_2(k', p', z') \, dg(z', z).$$

(6)

The right side represents the shadow value of cash balances, and the left side represents the marginal cost of external equity finance. To develop the intuition behind the optimal policy, we use the envelope condition to rewrite (6) as:

$$1 + (\lambda_1 - \lambda_2 e) \Phi_e = \frac{1 + r (1 - \tau_c)}{1 + r} \int (1 + (\lambda_1 - \lambda_2 e') \Phi'_e) \, dg(z', z).$$

(7)

Rewriting (6) as (7) makes it clear that without costly external finance, equation (7) holds as an inequality. In this case the tax penalty for saving implies that the firm never saves; that is, $p = p' = 0$. In contrast, in the face of costly external finance, if a firm saves a dollar today, it reduces the probability of having to issue new equity tomorrow. The firm continues to save just to the point where the gain from reducing future equity costs outweighs the tax penalty on saving. Inspection of (6) also reveals that optimal saving policy and optimal investment policy are clearly intertwined.

In some instances the fixed costs of external equity will be larger than the intramarginal gains from equity issuance. In these cases the firm is in a region of financial inertia in which it neither issues equity nor distributes funds to shareholders. Internal funds are the marginal source of funds and the firm saves any excess cash flows not used for positive NPV projects.

This discussion of the intuition for the value of cash in our model reveals a fundamental difference between our model and the model in Almeida, Campello, and Weisbach (2004) that goes beyond
the distinction between the finite horizon in their model and the infinite horizon in ours. In their three-period model capital depreciates completely between periods. Therefore, cash is the only way to transfer resources between periods. If a firm has insufficient resources to obtain the first-best investment policy, then if it gets a small increment to its income, it must save some in order to equate the marginal products of capital across time. This effect does not operate in our model. On a priori grounds a model with complete capital depreciation is unlikely to describe the data, given the small depreciation rates for different types of capital estimated by the Bureau of Economic Analysis. See Fraumeni (1997). We therefore retain the feature of partial capital depreciation each period.

II. Simulations

We solve the model numerically and investigate its implications for reduced-form regressions via simulation. We first describe the parameterization of our baseline simulation and explain the properties of optimal firm behavior. We then explain the experiments we perform on the model and the results of these experiments. We conclude by considering the empirical predictions given by the model and by examining the robustness of the model to our various simplifying assumptions.

A. Model Calibration

The profit function is given by $\pi (k, z) = zk^\theta$, in which we calibrate $\theta$ from the estimates of labor shares and mark-ups in Rotemberg and Woodford (1992, 1999). Their estimates, along with the assumptions of a Cobb-Douglas production function and a constant elasticity demand function, imply that $\theta \approx 0.75$. To specify a stochastic process for the shock $z$, we follow Gomes (2001) and assume that $z$ follows an $AR(1)$ in logs,

$$\ln (z') = \rho \ln (z) + v', \quad (8)$$

where $v' \sim N (0, \sigma_v^2)$. Our baseline parameter choices for $\rho$ and $\sigma_v$ are the averages of the estimates of these two parameters in Hennessy and Whited (2007): the serial correlation of the shock, $\rho$, is set at 0.66 and the standard deviation of the shock, $\sigma_v$, is set at 0.121.
We again follow Hennessy and Whited (2007) to parameterize the financing function, setting $\lambda_0 = 0.389$, $\lambda_1 = 0.053$, and $\lambda_2 = 0.0002$. These settings are from their estimates of the costs of external equity finance for large firms and are therefore conservative, lying only slightly above the figures for underwriting costs in Altinkilic and Hansen (2000). The number 0.389 in the data used by Hennessy and Whited (2007) implies a fee of $50,332 for the first million dollars of gross equity proceeds. We set the interest rate, $r$, equal to 4%, which lies between the values chosen by Hennessy and Whited (2007) and Gomes (2001).

To find values for the adjustment cost parameters, $c$ and $a$, we turn to Cooper and Haltiwanger (2006), who find that both convex and fixed costs of adjustment affect investment. From their estimates we set $c = 0.039$ and $a = 0.049$. We set the depreciation rate equal to 0.15, a figure approximately equal to the average in our data of the ratio of depreciation to the net capital stock.

Finally, to find a numerical solution we need to specify a finite state space for the three state variables. We let the capital stock lie on the points

$$\left[\bar{k} (1 - d)^{40}, \ldots, \bar{k} (1 - d)^{1/2}, \bar{k}\right].$$

We let the productivity shock have 25 points of support, transforming (8) into a discrete-state Markov chain using the method in Tauchen (1986). We let $p$ have 40 equally spaced points in the interval $[0, \bar{p}]$, in which $\bar{p}$ is set to $\bar{k}/2$. The optimal choice of $p$ never hits this upper bound.

We solve the model via iteration on the Bellman equation, which produces the value function $V(k, p, z)$ and the policy function $\{k', p'\} = h(k, p, z)$. In the subsequent model simulation, the space for $z$ is expanded to include 100 points, with interpolation used to find corresponding values of $V, k$, and $p$. The model simulation proceeds by taking a random draw from distribution of $z'$ (conditional on $z$), and then computing $V(k, p, z)$ and $h(k, p, z)$. We use these computations to generate an artificial panel of firms by simulating the model for 10,000 identical firms over 200 time periods, keeping only the last 20 observations for each firm.

B. Simulated Policy Functions

Before presenting our simulation results, we examine the economics behind the model by exploring the simulated policy function, $\{k', p'\} = h(k, p, z)$. We do so by plotting optimal cash flow,
investment (net of adjustment costs), saving, and distributions/equity issuance (net of issuance costs) as a function of \( z \) for three different \((k, p)\) pairs: low \( k / \text{medium } p \), medium \( k / \text{medium } p \), and high \( k / \text{medium } p \). By high, medium, and low we mean the maximum, median, and minimum values that \( k \) and \( p \) take in the baseline simulation. We focus on the medium \( p \) case for brevity, but we discuss the high and low \( p \) cases briefly below. Cash flow is defined precisely as \((1 - \tau_c) \pi(k, z) / k^*, \) investment as \(((k' - (1 - d)k) - A(k, k')) / k^*, \) saving as \( (p' / (1 + r (1 - \tau_c)) - p) / k^*, \) and net distributions/equity issuance as \( (e(k, k', p, p', z) + \phi(e(k, k', p, p', z))) / k^*, \) where \( k^* \) is the steady-state level of the capital stock. We deflate our variables of interest by \( k^* \) for the three differently sized firms to facilitate comparisons between them.

Figure 1 contains these plots, with the three panels depicting, respectively, a small, medium, and large firm. In all three panels cash flow naturally rises with the \( z \) shock. These cash flows are distributed differently, however, depending on the size of the firm.

[Figure 1 about here.]

For the small firm investment rises smoothly with cash flow. Despite the presence of adjustment costs, the capital stock is so low and the marginal product of capital so high that a higher value of \( z \) almost always means more investment.

In contrast, the behavior of saving is nonmonotonic. Although the small firm always saves, saving initially rises with \( z \) and then falls. This hump-shaped pattern reflects income and substitution effects. To define these effects, we note that when \( z \) rises, both capital productivity and cash flow rise; that is, the firm’s value function both becomes steeper and shifts upward. We define the income effect as \( \partial p / \partial z \) along the surface where \( V_1(k, p, z) \) and \( V_2(k, p, z) \) are unchanged. In words, the income effect captures the upward shift and thereby isolates the effect of the extra income generated by \( z \), holding current and future productivity of capital and cash constant. We define the substitution effect as \( \partial p / \partial z \) minus the income effect. The substitution effect captures the changed steepness of the value function and thereby isolates the effects of current and future changes in the marginal productivities of cash and capital.
We now use these definitions to explain the hump-shaped pattern. As \( z \) rises, capital productivity rises and the firm expects productivity to revert to its mean slowly because of the positive serial correlation in \( z \). The substitution effect implies that the firm saves less because it wants to shift some of its liquid assets into physical assets that have become relatively more productive. The income effect implies that the firm saves more as it increases both physical and liquid assets, both of which continue to have value as \( z \) rises. The income effect dominates for low levels of \( z \), but its strength is limited, and the substitution effect dominates for high levels of \( z \).

In the model distributions/equity issuance are a residual. For a small firm the marginal product of capital is sufficiently high that it is optimal for the firm to issue equity and pay issuance costs regardless of the level of the productivity shock.

The medium-sized firm behaves quite differently. First, the optimal investment rule displays substantial inertia. For low levels of \( z \) the firm sells capital, but for intermediate and high levels of \( z \) the firm invests. Investment initially rises with \( z \), but then flattens out, rising once again when \( z \) is high. Physical adjustment costs cause the stagnation, and the fixed equity issuance cost causes the jump, as the firm goes from making distributions to raising equity. Saving also behaves differently in the medium-sized firm. Saving is always negative and always decreases with \( z \) and with cash flow because the medium firm does not tap external finance as often as the small firm. Therefore, cash is less valuable, which leads the substitution effect to dominate the income effect. This negative correlation between cash flow and saving is crucial for understanding the saving sensitivity results that follow. Finally, for low levels of \( z \), the firm distributes excess funds to shareholders because the benefits of investing do not outweigh the costs of issuance. However, if \( z \) rises to a sufficiently high level, the benefits from investing start to outweigh issuance costs, and the firm issues equity.

The large firm, not surprisingly, sells capital for low to intermediate levels of \( z \) because the marginal product of capital is low. Although investment eventually becomes positive as \( z \) rises, the presence of adjustment costs combined with the low marginal product of capital cause the rate of investment to level out for very high levels of \( z \). Saving initially declines with \( z \) because of the substitution effect, which operates even though the firm is disinvesting because the marginal product of capital always rises with \( z \) regardless of optimal investment policy. However, dissaving
flattens for high levels of $z$ because the model does not allow for negative cash (i.e., debt).

For brevity we have not plotted the optimal saving and investment rules for firms with either low or high cash balances. In general, the same effects operate, although two further patterns are of interest. First, the income effect on saving dominates for firms with low cash balances. This result makes sense because the value of cash is especially high when cash balances are low. Therefore, the substitution effect that arises from the relative increase in capital productivity is small. Second, and conversely, the substitution effect on saving dominates for firms with larger cash balances.

C. Experiments

With the model intuition in hand we now turn to our simulation results. We investigate two ways in which the model’s parameters affect the firm’s cash and saving policies. We first consider how the parameters affect the level of cash as a fraction of assets, which is defined in our model as the average of $p/k$ over all of the observations in the simulated panel. We then examine how the parameters affect a measure of saving behavior that first appears in Almeida, Campello, and Weisbach (2004). Dubbed “the cash flow sensitivity of cash,” this measure is defined in our model as the regression coefficient, $\alpha_1$, in the following regression:

$$
\frac{p' - p}{k} = \alpha_0 + \beta \frac{V(k, p, z)}{k} + \alpha_1 \frac{\pi(k, z)}{k} + \alpha_2 \ln(k) + u,
$$

where $\alpha_0$, $\alpha_1$, $\alpha_2$, and $\beta$ are regression coefficients and $u$ is a regression disturbance, which in our simulations is, by definition, orthogonal to the regressors. This regression comes directly from Almeida, Campello, and Weisbach (2004), and we estimate it with all of the observations in the simulated panel. The intent is to understand better the economics behind this reduced-form regression.

In thinking about the following results, it is crucial to distinguish cash levels ($p$) and saving ($p' - p$) from the saving sensitivity ($\alpha_1$), which is the partial correlation between cash flow and saving, holding $q$ and $k$ constant. For example, a negative sensitivity does not imply that $p' - p < 0$, that is, that the firm always dissaves; nor does $\alpha_1 < 0$ imply a negative simple correlation between cash flow and saving. Indeed, in most of our simulations, cash as a fraction of assets is small and positive, and the simple correlation between cash flow and saving is positive, even though $\alpha_1 < 0$. 
We examine the sensitivity of our two gauges of cash policy \((p/k\) and \(\alpha_1\)) to eight key model parameters: the standard deviation and serial correlation of log profits, \(\sigma_v^2\) and \(\rho\); the three equity cost parameters, \(\lambda_0\), \(\lambda_1\), and \(\lambda_2\); the curvature of the profit function, \(\theta\); and the fixed and quadratic adjustment cost parameters, \(c\) and \(a\). In each of the following experiments, we set all but one of the parameters equal to their baseline levels, allowing the free parameter to range within a given interval. We allow \(\theta\) to range from 0.6 to 0.9, \(\rho\) from -0.8 to 0.8, \(\sigma_v\) from 0.05 to 0.2, \(\lambda_0\) from 0 to 0.8, \(\lambda_1\) from 0 to 0.1, \(\lambda_2\) from 0 to 0.0004, \(c\) from 0 to 0.8, and \(a\) from 0 to 0.1.

Figure 2 illustrates the sensitivity of the optimal choice of \(p/k\) to the model parameters. We first examine the parameters that govern the stochastic shock process. The first panel shows a u-shaped relation between the serial correlation of income, \(\rho\), and cash holdings. For both highly positively and highly negatively correlated shocks, the firm holds high cash balances, choosing lower balances if the shocks are less highly correlated. Two separate effects explain this result. First, as \(\rho\) increases, the firm invests in larger amounts because a positive productivity shock signals not only that capital is productive today, but also that it will continue to be productive. The firm therefore wants higher cash balances to lower the probability of needing external finance when it makes these large investments. Second, the higher the serial correlation of an \(AR(1)\) process, the higher its variance. If the firm faces an uncertain environment, it expects to tap external finance more often, and it holds higher cash balances. Both effects operate in the same direction for high levels of \(\rho\), but they tend to offset each other for levels near zero. For levels of \(\rho\) far below zero, the second effect dominates. The intuition about the effect of uncertainty is also evident in the second panel, which depicts a positive relation between cash holdings and \(\sigma_v\), the standard deviation of the innovations to \(\ln(z)\). The increase in cash accompanying an increase in \(\sigma_v\) also has a real options interpretation in which a higher variance leads to a higher option value of cash balances.

[Figure 2 about here.]

The third through fifth panels illustrate the effects of each of the external finance parameters on cash holdings. Not surprisingly, the third and fourth panels show that cash increases with
the fixed and linear components of the external finance function, \( \lambda_0 \) and \( \lambda_1 \), because the value of financial flexibility increases as external finance becomes more costly. However, the relation shown in the fifth panel between the quadratic component, \( \lambda_2 \), and cash holdings is flat. With \( \lambda_0 \) and \( \lambda_1 \) set to their baseline levels, the effect of \( \lambda_2 \) is of second-order importance. These results mirror those in the three-period model of Almeida, Campello, and Weisbach (2004), who produce a partial derivative of cash with respect to internal funds that is positive for a financially constrained firm, and zero otherwise.

Finally, the sixth through eighth panels display the effects of technology. The sixth panel reveals a hump-shaped relation between production function curvature (\( \theta \)) and cash. Two different economic forces create this pattern. First, as \( \theta \) rises, the production function becomes flatter, and the average size of desired investments rises. The firm holds more cash because large investments imply a greater likelihood of needing external finance. Second, as \( \theta \) rises, the firm is less likely to have to tap external finance because a higher \( \theta \) implies that a given capital stock can create more internal revenue, and the firm therefore needs to hold less cash. The first effect is stronger for lower levels of \( \theta \), and the second effect is stronger for higher levels of \( \theta \). The seventh panel shows that cash holdings increase with the fixed cost of adjustment. This effect occurs because higher fixed adjustment costs lead to larger investments that occur less frequently. The firm then uses episodes of inaction to accumulate cash, which acts to lower the probability of the firm having to tap external finance when it does invest. Finally, the eighth panel shows that convex adjustment costs have the opposite effect on cash holdings. As \( \alpha \) increases, the firm makes smaller investments more often, is therefore less likely to have to tap external finance, and holds less cash.

These results on the level of cash balances reassuringly confirm those in Gamba and Triantis (2006), in particular, their results on the effects of uncertainty and the cost of external finance. Our results on cash levels are also useful in providing intuition for the main focus of this paper, which is not cash levels per se, but the propensity to save.

Figure 3 is analogous to Figure 2, except that it depicts how the model parameters affect our second measure of cash policy—the coefficient \( \alpha_1 \) in (9); that is, the sensitivity of saving to cash flow, holding Tobin’s \( q \) and size constant. A quick glance at the figure reveals that for almost all
model parameterizations this sensitivity is negative. Because this result is the opposite of that produced by the model in Almeida, Campello, and Weisbach (2004), we examine why. The answer partly lies in the difference between income and substitution effects. In our model, when a firm receives a positive income shock, its cash flow rises. In addition, if the shock is not too transitory, both the current and future productivity of capital rise. A substitution effect implies that the firm wants to transform its financial assets into relatively more productive physical capital, that is, dissave. An income effect implies that the firm wants to use some of its increased cash flow to increase cash balances, thereby lowering the probability that it will have to use costly external funds to finance future investment. In a regression of saving on $q$ and cash flow, both $q$ and cash flow contain information about capital productivity. Because cash flow, $\pi(k,z)$, is driven in large part by $z$, and because, as demonstrated in Figure 1, $z$ drives the substitution effect, the negative coefficient on cash flow reflects the negative substitution effect. The coefficient on $q$ gives the partial effect on savings of changes in $q$, holding $\pi(k,z)$ and $k$ constant. It must therefore be driven by changes in the only remaining state variable, which is the current cash level, $p$. In particular, $q$ captures the value of both physical assets and cash, whereas cash flow only captures the value of physical assets. We therefore conjecture that $q$ reflects the increased value of cash following a positive productivity shock and therefore captures the income effect. In contrast, in the Almeida, Campello, and Weisbach (2004) model the sensitivity of saving to cash flow is defined as the response of cash holding to an exogenous increase in the firm’s endowment, which does not affect capital productivity. Therefore, their model produces a positive sensitivity in part because it only captures an income effect. As mentioned above, their model also produces a positive sensitivity because capital depreciates completely between periods.7

We now turn to a more detailed discussion of Figure 3. The first panel shows our most interesting simulation result, which is the effect of the serial correlation of the shock process ($\rho$) on the cash flow sensitivity ($\alpha_1$). If $\rho$ is highly negative, $\alpha_1$ is large and positive; if $\rho$ is highly positive, $\alpha_1$ is large and negative; and if $\rho$ is near zero, $\alpha_1$ is near zero. If profits are negatively serially correlated,
then a positive shock implies an expected productivity decline, which in turn implies a low need for external finance. This income effect promotes dissaving. A stronger substitution effect, however, promotes saving because the expected productivity decline prompts the firm to funnel cash flow into liquid assets and distributions rather than into investment. On the other hand, if the firm faces highly positively correlated income shocks, then the income and substitution effects operate in the opposite direction, and, controlling for $q$, the firm dissaves when it experiences a positive shock.

The second panel illustrates the effect of the shock variance. The saving sensitivity is always negative, but becomes less so as $\sigma_v$ increases. As the firm’s environment becomes more uncertain, its level of cash increases, but it also becomes more reluctant to change its cash holdings aggressively in response to shocks, which convey little information in an uncertain environment. We emphasize that because $\alpha_1$ depends on the variability and autocorrelation of $z$, any observed cross-sectional variability in $\sigma_v$ and especially in $\rho$ renders $\alpha_1$ a poor financial constraint indicator.

The third through fifth panels examine the effects of the cost of external finance. The patterns evident in these panels mirror those in the corresponding panels in Figure 2. In all cases, as the cost of external finance increases, the level of cash increases, and the sensitivity of saving to cash flow becomes more negative. Because a firm with a high cost of external finance optimally holds a high level of cash, it can respond to shocks more aggressively by changing its cash balances. For example, if a positive profit shock hits a firm with a high level of cash, it will dissave a great deal in order to invest. An otherwise identical firm with a low level of cash cannot dissave as much.

Finally, we examine technology. The saving sensitivity becomes more negative as $\theta$ increases, that is, as the production function becomes flatter. With a flat production function positive shocks induce large desired increases in the capital stock, and the firm dissaves to fund these investments. The saving sensitivity also becomes more negative as fixed adjustment costs increase because the firm optimally invests in large amounts. Similarly, the saving sensitivity becomes less negative with the quadratic adjustment cost because the firm optimally invests in small amounts.

The preceding arguments are valid at points in time in which the firm is actively adjusting its capital stock. During periods of inaction, the sensitivity of saving to cash flow is positive.
because the firm funnels at least part of its cash flow into cash holdings. Under almost all model parameterizations the firm adjusts more often than it remains inactive. The observations in which the saving sensitivity is negative therefore outweigh those in which it is positive, and average sensitivity is negative.

The frequent adjustment in our model sets it apart from models of dynamic capital structure with adjustment costs. For example, in Fischer, Heinkel, and Zechner (1989), the firm adjusts its asset and liability composition infrequently. As pointed out in Strebulaev (2007), empirical predictions from this sort of model cannot be based on firm behavior at points in time at which the firm is inactive. The frequent adjustment in our model allows us to sidestep this critique. Frequent adjustment also explains why corporate propensities to save can be negative even though personal propensities to save are typically positive. Although consumers dissave when they purchase durables, these events are infrequent, and because consumers save out of income at other times, average personal saving propensities are therefore positive.

Because we are interested in the effect of measurement error in observed Tobin’s $q$ in our data, we conduct a further simulation in which we introduce an additive $i.i.d.$ measurement error to true Tobin’s $q$: $V(k,p,z)/k$. Measurement error biases the positive coefficient on $V(k,p,z)/k$ downward, but it biases the coefficient on cash flow $(\pi(k,z)/k)$ upward because of the strong positive correlation between $\pi(k,z)/k$ and $V(k,p,z)/k$. We find in the baseline simulation that the error variance needs to be at least eight times as large as the variance of $V(k,p,z)/k$ to reverse the initially negative sign of the coefficient on cash flow. This result is in accord with the empirical results that follow inasmuch as we also estimate a measurement error variance much higher than the variance of true unobserved Tobin’s $q$.

In sum, these experiments highlight three important pieces of economic intuition. First, corporate saving depends not only on the firm’s financial environment, but also on its technology. Second, variation in capital productivity is critical for our results, because a model cannot capture the firm’s desire to substitute capital for cash if the marginal product of capital is constant. Third, although the levels and changes in cash are related, a high cash level does not necessarily imply a high positive sensitivity of saving to cash flow; nor does a low cash level imply a low sensitivity. We
emphasize again that this distinction is impossible to uncover in a model with a one-period saving
decision because the change in cash cannot be distinguished from the level of cash.

D. Empirical Predictions

The simulations in Figure 3 delineate the four central empirical predictions we test. First, the
sign of $\alpha_1$ in the regression (9) should be negative. Second, $\alpha_1$ should increase in absolute value
with the cost of external finance. Third, $\alpha_1$ should decrease in absolute value with $\sigma_v$. Fourth, $\alpha_1$
should increase in absolute value with $\rho$. We consider the relation between $\alpha_1$ and both production
function curvature and adjustment costs only in robustness checks because available proxies for
curvature and adjustment costs are weak.

Testing these predictions in this manner forms a strong link between the theory and its test
because the form of the real-data test is identical to the form of the simulated-data theoretical
prediction. Further, because the prediction encompasses the entire regression specification, and
because the error term in a linear regression (linear projection) is by definition orthogonal to the
right-hand-side variables, testing whether $\alpha_1 < 0$ in this manner avoids the usual simultaneity
problems that plague regressions in corporate finance. This type of prediction also has precedents
in Whited (2006) and Caggese (2007). Finally, this type of a link between theory and its tests
is fundamentally different from testing model predictions that take the form of the sign of a par-
tial derivative, because partial derivatives provide incomplete guidance on the entire regression
specification.

E. Model Robustness

The model is intentionally sparse to highlight intuition. To assuage concerns that our results
are artifacts of the model’s simplicity, in this section we add several more realistic features to the
model to examine the robustness of our result of a negative saving propensity.

Our first set of robustness checks focuses on specific features omitted from our baseline model.
First, in the baseline model the firm does not have access to a credit line. When we add riskless
short-term debt that is secured by the capital stock, as in Hennessy and Whited (2005), the saving
propensity of -0.40 in the baseline simulation drops in absolute value to -0.22. Our results are
attenuated but not erased because the upper limit to the credit line causes cash to retain its value as a tool to avoid costly external finance. Second, the baseline firm does not smooth distributions to shareholders. To address this possibility, we penalize the firm by the amount of the linear equity issuance cost for every dollar that its distributions fall below the average level of distributions in the baseline simulation. This model feature produces increased cash hoarding because the firm wants to avoid missing a distribution. This higher cash cushion leads to a more negative propensity to save of -0.54. Third, the baseline firm has no fixed costs of production, which could, for example, represent the tendency of young firms to burn profits. We add a cost of production equal to 0.4\(k\), in which 0.4 is the approximate ratio of selling, general, and administrative expenses to assets in our U.S. sample. This addition to the model produces less cash holdings relative to our baseline model because the firm has smaller profits to funnel into liquid assets. Accordingly, the saving propensity drops in absolute value to -0.12. Fourth, we allow the firm to issue risky debt, which we model exactly as in Hennessy and Whited (2007). This approach necessitates the exclusion of any physical adjustment costs. In this case the firm hoards more cash than in the baseline simulation in order to avoid default, and the saving propensity rises in absolute value to -0.59. We choose to exclude these features from our baseline model because they do not change any qualitative simulation outcomes, and a simple structure is important for lending intuition to our empirical work.

Finally, because our model contains only one source of uncertainty, productivity shocks and cash flow are almost perfectly correlated. Therefore, the dissaving that occurs with a positive productivity shock is necessarily accompanied by a rise in cash flow. To ascertain whether our finding of a negative saving propensity is hard-wired by this feature of our model, we allow the net revenue function to take the form \(zk^\theta - \eta k^*\), in which \(k^*\) is the steady state capital stock for a frictionless version of this model and \(\eta\) is a normally distributed, zero-mean, \(i.i.d\). random variable with a variance equal to that of the \(z\) shock. This new cost shock takes four points of support, and its transition matrix is given by the method in Tauchen (1986). Not surprisingly, decoupling cash flows from the productivity shock produces a much smaller (in absolute value) propensity to save. However, unless the variance of the cost shocks is twice the variance of the productivity shock, the saving propensity remains negative. Further, even with a positive saving propensity,
the serial correlation and variance of both types of income shocks still have important effects on
the saving propensity. Finally, Smets and Wouters (2007) estimate on a macroeconomic level that
the variances of cost and productivity shocks are comparable in magnitude. Although this result
indicates that a low correlation between cash flow and productivity is unlikely to be a pervasive
phenomenon, cross-sectional heterogeneity in this correlation remains an empirical issue that we
explore below.

The second set of robustness checks relates to the connection between estimating (9) with
simulated data and with real data. The cross sections generated by the model contain 10,000
identical firms over 20 time periods. This simulated cross-section asymptotically generates the
same results as a single time series with 200,000 observations. In contrast, our real data contain
heterogeneous firms. Therefore, unless a simulated panel of heterogeneous firms can generate a
negative sensitivity, the connection between the theory and its tests becomes tenuous.

Adding heterogeneity to the simulated sample should produce a positive sensitivity only if
most simulated firms come from a model that generates a positive sensitivity. The predominantly
negative sensitivities seen in Figure 3 indicate that a positive sensitivity is only likely to arise
in a panel that is heterogeneous in the serial correlation parameter, $\rho$. Indeed, we find negative
sensitivities when we add heterogeneity by varying the shock standard deviation ($\sigma$), issuance costs
($\lambda_0, \lambda_1, \lambda_2$), returns to scale ($\theta$), and fixed and smooth adjustment costs ($c, a$). In contrast, when
we divide the cross section into 10 groups with values of $\rho$ equally spaced between -0.8 and 0.8,
we find a small positive sensitivity of 0.008. This result begs the question of the cross-sectional
distribution of the serial correlation of income in our real data. To answer the question, we use
our U.S. data to estimate a first-order autoregression of operating income either firm-by-firm or
industry-by-industry, in which an industry is defined at the three-digit level. In the firm-level
autoregressions we find that only 7.1% of our firms have negatively serially correlated income.
In the industry-level autoregressions we never find negatively serially correlated income. To add
heterogeneity in serial correlation that approximates the situation in our data set, we rerun our
simulation with the cross section divided into 10 groups with values of $\rho$ that approximate the
firm-level cross-sectional distribution of income serial correlation in our real data. In this case we
do find a negative sensitivity.

It is also interesting to see whether we can generate a positive sensitivity when we add heterogeneity to the sample by varying parameters not examined in Figure 3. We look at the discount factor \((\beta)\), the rate of capital depreciation \((d)\), and the drift of the shock process, the latter of which we model by adding an intercept to (8). In none of these cases are we able to generate a positive sensitivity. Decreasing the depreciation rate or increasing the discount rate reduces the average size of investments, the need for external finance, and cash levels. The saving propensity remains negative but decreases in absolute value. Increasing either the depreciation rate or the discount factor produces the opposite effects. The driving force behind these results is the interplay between stocks and flows. Firms with a high stock of cash have more leeway with which to respond to shocks, and therefore have a more negative sensitivity. Changing the drift of the \(z\) process alters the average size of investments but leaves the saving propensity negative. As a final note, we can generate a positive sensitivity by turning off the smooth adjustment costs and multiplying the fixed adjustment costs by a factor of 10. In this simulation the firm invests sporadically in large spikes, accumulating cash during the periods of inactivity.

In sum, our result of a negative propensity to save is remarkably robust. We only find three instances in which we can generate a positive saving propensity: with negatively serially correlated productivity shocks, with high variance cash flow shocks that are decoupled from productivity shocks, and with high fixed costs of adjustment. We have provided evidence that the first situation is not relevant empirically, and we examine the empirical relevance of the other two situations below.

III. Data and Summary Statistics

We obtain data on U.S. nonfinancial firms from the 2007 Standard and Poor’s Compustat industrial files. These data constitute an unbalanced panel that covers 1972 to 2006. We also draw data from Standard and Poor’s Compustat Global Issue and Industrial/Commercial for Canada, France, Germany, Japan, and the United Kingdom. These data also constitute an unbalanced panel but only cover 1994 to 2005 because Global Vantage does not report data on all firms for 2006.
To select the sample, we delete firm-year observations with missing data and for which total assets, the gross capital stock, or sales are either zero or negative. Then for each firm we select the longest consecutive times series of data and exclude firms with only one observation. Finally, we omit all firms whose primary SIC code is between 4900 and 4999, between 6000 and 6999, or greater than 9000, because our model is inappropriate for regulated, financial, or quasi-public firms.

We define variables from Global Industrial/Commercial as follows: book assets is item 89; investment is item 193; operating income is item 14; cash flow is item 11 plus item 32; and cash is item 60. The numerator of the market-to-book ratio is the sum of the market value of equity (item 3 times item 13 in Global Issue) and book assets minus the book value of equity (item 105 plus item 135), and the denominator is book assets. For our U.S. data from Compustat we define book assets as item 6, operating income as item 13, cash flow as item 14 plus item 18, and cash as item 1. The numerator of the market-to-book ratio is the sum of the market value of equity (item 199 times item 25) and total book assets minus the book value of equity (item 60 plus item 74). The denominator is book assets. In our regressions we scale saving and cash flow by book assets. We delete the top and bottom 1% of our regression variables.

Summary statistics are in Table I. First, the means of Tobin’s q (market to book) are much larger than the medians. As explained below, this skewness is essential for identifying our econometric model. The average ratio of cash to assets ranges from 0.09 to 0.16. These averages hide a sharp increase in this ratio in the last 10 years. For example, in the United States the median has doubled over this time period. Finally, the average number of observations per year is in the last column. Both the United States and Japan have on average more than 2,500 observations per year, whereas Canada, France, and Germany each have fewer than 400. Because our estimation method requires a great deal of data, the variation in sample size across countries manifests itself in the precision of the estimates we obtain from each country.

[Table I about here.]

IV. Estimation
In our model we can perfectly observe $V(k, p, z)/k$. In contrast, in our data we use Tobin’s $q$ as a proxy for $V(k, p, z)/k$. As explained in Erickson and Whited (2000), Tobin’s $q$ is an imperfect proxy, and we must therefore treat the ensuing measurement error. This section outlines the method we use. It then presents the results from applying this technique to the data.

A. Methodology

We use the estimators in Erickson and Whited (2000, 2002), which employ the structure of the classical errors-in-variables model. Applied to a single cross section, this model can be written as

\[ y_i = w_i \alpha + \chi_i \beta + u_i, \quad (10) \]
\[ x_i = \gamma + \chi_i + \varepsilon_i. \quad (11) \]

In our application $y_i$ is the ratio of the change in cash to assets, $\chi_i$ is the true $q$ of firm $i$, $x_i$ is an estimate of its true $q$, and $w_i$ is a row vector of perfectly measured regressors, whose first entry is one, whose second entry is the ratio of cash flow to assets, and whose third entry is the natural log of total assets. The regression error, $u_i$, and the measurement error, $\varepsilon_i$, are assumed to be independent of each other and of $(w_i, \chi_i)$, and the observations, $(\varepsilon_i, u_i, w_i, \chi_i)$, $i = 1, \ldots, n$, are i.i.d. The intercept, $\gamma$, in (11) allows for systematic bias in the measurement of $\chi_i$. We do not require any assumptions about the temporal dependence or independence of $(\chi_i, w_i, u_i, \varepsilon_i)$.

To derive a set of tractable moment conditions to be estimated by GMM, we reexpress (10) and (11) in terms of the residuals from the regressions of $y_i$, $x_i$, and $\chi_i$ on $w_i$. Let $(\hat{y}_i, \hat{x}_i, \hat{\chi}_i)$ be the residuals from the linear projection of $(y_i, x_i, \chi_i)$ on $w_i$. Then (10) and (11) can be written as

\[ \hat{y}_i = \beta \hat{\chi}_i + u_i \]
\[ \hat{x}_i = \hat{\chi}_i + \varepsilon_i. \]

If we square (12), multiply the result by (13), and take expectations of both sides, we obtain

\[ E(\hat{y}_i^2 \hat{x}_i) = \beta^2 E(\hat{\chi}_i^3). \quad (14) \]

Analogously, if we square (13), multiply the result by (12), and take expectations of both sides, we obtain

\[ E(\hat{y}_i \hat{x}_i^2) = \beta E(\hat{\chi}_i^3). \quad (15) \]
As shown in Geary (1942), if $\beta \neq 0$ and $E(\hat{\chi}_i^3) \neq 0$, dividing (14) by (15) produces a consistent estimator for $\beta$ that equals $E(\hat{y}_i^2 \hat{x}_i) / E(\hat{y}_i \hat{x}_i^2)$. The assumptions $\beta \neq 0$ and $E(\hat{\chi}_i^3) \neq 0$ are necessary for identification because one cannot divide by zero. These assumptions can be tested via the null hypothesis that $E(\hat{y}_i^2 \hat{x}_i) = 0$ and $E(\hat{y}_i \hat{x}_i^2) = 0$. We refer to this test hereafter as an identification test. It is a useful regression diagnostic inasmuch as it provides information as to whether the coefficient estimates are reliable. For example, if $\hat{\chi}_i$ is near normally distributed, the identification test will not produce a rejection and the coefficient standard errors will be large. It is worth noting that we do not use this test to select our samples.

This estimator is a third-order moment estimator. The innovation in Erickson and Whited (2000, 2002) consists of combining the information in moment equations of orders 2 through 7 via GMM to obtain a more efficient estimator for $\beta$. It is possible to estimate many interesting quantities besides $\beta$. For example, the coefficient of determination ($R^2$) of (11), denoted as $\tau^2$, is given by

$$
\tau^2 = \frac{\mu_y \var(w_i) \mu_x + E(\hat{\chi}_i^2)}{\mu_y \var(w_i) \mu_x + E(\hat{\chi}_i^2) + E(\hat{\varepsilon}_i^2)}.
$$

(16)

It is a useful index of measurement quality for our proxy for unobservable $V(k, p, z)/k$. A value close to one indicates a nearly perfect proxy, and a value close to zero indicates a nearly worthless proxy. An exactly analogous formula provides the measurement-error consistent estimate of the $R^2$ of (10). We can also estimate the coefficient vector $\alpha$, which can be recovered by the identity

$$
\alpha \equiv \mu_y - \beta \mu_x.
$$

(17)

in which $(\mu_y, \mu_x)$ are the vectors of coefficients in the population projection of $(y_i, x_i)$ on $w_i$. This identity is useful for understanding why measurement error in Tobin's $q$ biases the cash flow coefficient even when saving is on the left side of the regression. To simplify the explanation, we isolate $\alpha_1$, the cash flow coefficient, and rewrite the second element of the vector equation (17) as

$$
\alpha_1 \equiv \mu_{1y} - \beta \mu_{1x}.
$$

(18)

The first term in (18), $\mu_{1y}$, is the coefficient on cash flow obtained by regressing saving on only cash flow and firm size. The second term represents the extent to which $\mu_{1y}$ changes when one
controls for true unobservable Tobin’s $q$. This term clarifies how measurement error in Tobin’s $q$ biases the cash flow coefficient. It is well known that measurement error biases $\beta$ downward, and the amount of bias is approximately proportional to $\tau^2$. If $\mu_{1x} = 0$, then this downward bias has no effect on the coefficient on cash flow. However, if $\mu_{1x} \neq 0$, then a downward-biased $\beta$ can affect the cash flow coefficient, $\alpha_1$. Recall that $\mu_{1x}$ is the slope coefficient on cash flow one obtains from regressing observable Tobin’s $q$ on cash flow and size. Because Tobin’s $q$ and cash flow are positively correlated, and because the variance of Tobin’s $q$ is much greater than the variance of cash flow, $\mu_{1x}$ can be large. In our application it ranges from one to five. Therefore, a small downward bias in $\beta$ can cause a large upward bias in the OLS estimate of the cash flow coefficient. Finally, it is crucial to note that the bias in the coefficient $\alpha_1$ does not depend on the correlation between the left-hand-side variable and the right-hand-side variables. Instead, what matters is both the measurement quality of Tobin’s $q$ and the covariance matrix of the regressors. Therefore, measurement error in Tobin’s $q$ can bias other regression coefficients regardless of the left-hand-side variable.

Riddick and Whited (2008) presents two sets of simulations that evaluate these estimators. The first set consists of Monte Carlo simulations that assess the finite sample performance of these estimators on data closely resembling our own. Of particular interest in these simulations are the tests of the null hypothesis that the coefficient on cash flow equals its true value. The actual sizes of many of these tests are tiny relative to their nominal sizes because the GMM estimates of the standard errors are too large. Also of interest is the large upward bias in the OLS coefficient on cash flow, and the small downward bias in some, but not all, of the GMM estimates of the cash flow coefficient. This latter bias is at most 10% and therefore cannot account for our empirical results that follow.

The second set of simulations uses our theoretical model to generate data on heterogeneous firms, which we then use to examine how the estimators perform when the data exhibit heterogeneity that is not captured by our errors-in-variables model. We find that the estimators correctly produce cash flow sensitivities that are more negative for constrained than for unconstrained firms, as well as sensitivities that are less negative for high uncertainty firms than for low uncertainty firms.

Because these estimators can only be applied to samples that are arguably $i.i.d.$, we estimate
(10) and (11) for each cross section of our unbalanced panel and then pool the yearly estimates via the procedure in Fama and MacBeth (1973). We do not include firm fixed effects for four reasons. First, when we compare the results from running fixed effects OLS to those from using OLS with the Fama-MacBeth approach, we find almost identical coefficient estimates. Second, when we do a standard Hausman test to determine whether a potential fixed effect is correlated with the regressors, we cannot reject the null of no correlation. This result suggests that the within-firm variation in saving and $q$ mirrors the cross sectional variation. Third, including fixed effects reduces a great deal of the skewness and kurtosis that identify the slope coefficients. The resulting model therefore passes the GMM identification test less frequently. Fourth, the GMM estimates are qualitatively similar, but more unstable, when we include fixed effects. In sum, although it is somewhat unconventional not to control for fixed effects when one has panel data available, we do not because doing so affects our results little and because we do not wish to use up valuable degrees of freedom or remove interesting data variation.

Recently, Petersen (2007) reemphasizes that Fama-MacBeth standard errors can produce inflated $t$-statistics in panel data. Further, our lack of restrictions on the time series properties of $(\chi_i, u_i, \varepsilon_i)$ opens the door for the finite-sample critical values the Fama-MacBeth $t$-statistics to be much higher than the nominal critical values. We deal with this issue by using the bootstrap in Hall and Horowitz (1996) to calculate the finite-sample distribution of the $t$-statistics produced with the Fama-MacBeth standard errors. The unit of observation for resampling is the firm. Interestingly, we find that many of these bootstrapped critical values are only slightly higher than the asymptotic critical values, although in several instances we do find bootstrapped critical values as high as 5 for a nominal 5% two-sided $t$-test, especially in the case of the GMM estimates of the coefficient on $\chi_i$. 

27
B. Results

Table II presents the Fama-MacBeth results from estimating (10) via OLS and from estimating (10) and (11) via GMM for each of our six countries. The left panel shows the OLS results, and the right panel shows the GMM results. We report the OLS estimate of the regression $R^2$ in column 3, and the measurement-error consistent GMM estimate of the regression $R^2$ in column 6. Column 7 contains the estimate of our index of measurement quality: $\tau^2$. Standard errors are in parentheses below each parameter estimate. Asterisks and daggers mark the parameter estimates whose $t$-statistics exceed, respectively, the 5% bootstrapped and 5% asymptotic critical values.

For each country we test our first prediction that the coefficient on cash flow is negative in a regression of saving on $q$, cash flow, and size. Our OLS results corroborate earlier findings that the coefficients on both Tobin’s $q$ and cash flow are positive for all countries, as in, for example., Almeida, Campello, and Weisbach (2004) and Khurana, Martin, and Pereira (2006). However, when we apply the Erickson and Whited estimators to correct for measurement error in $q$, the results change. We find negative coefficients on cash flow in all six countries, and all are statistically significant according to our bootstrapped critical values. These results correspond to our model simulation results for firms that have positively serially correlated income processes. The effect of treating measurement error can also be seen in the GMM coefficients on $q$, which are from seven to fifteen times as high as their OLS counterparts in the different countries. This result can be explained by the attenuation bias in the classical errors-in-variables model, which in this case is very large because of the low estimates for $\tau^2$; that is, low estimated measurement quality of observed Tobin’s $q$. Finally, correcting for measurement error increases the regression $R^2$ substantially because measurement error obscures the contribution of true Tobin’s $q$ to the variation in saving.

The flip in the sign of the cash flow coefficient can be understood as follows. First, recall the identity given by (18): $\alpha_1 \equiv \mu_{1y} - \beta \mu_{1x}$, where $\mu_{1y}$ is the coefficient from regressing saving on cash
flow and $\mu_{1x}$ is the coefficient from regressing Tobin’s $q$ on cash flow. On average in our regressions $\mu_{1y}$ is about 0.15, and $\mu_{1x}$ is about 2. The biased OLS estimates of $\beta$, which hover around 0.02, therefore produce a positive cash flow coefficient when plugged into the above identity with these values for $\mu_{1y}$ and $\mu_{1x}$. In contrast, the consistent GMM estimate of $\beta$, which hovers around 0.2, produces a negative cash flow coefficient when plugged into this identity.

What is the intuition behind this econometric result? First, because $\mu_{1y}$ is approximately the simple correlation between saving and cash flow, and because the OLS estimate of $\beta$ is biased downward severely, the OLS estimate of $\alpha_1$ only picks up this simple correlation. A positive OLS coefficient makes sense in that on average companies should save part of cash flow shocks and then invest the rest or return it to shareholders. Conversely, our negative GMM coefficients do not imply that firms dissave more than the amount of a cash flow shock, because these coefficients only pick up the partial correlation between saving and cash flow, Tobin’s $q$ and size held constant. Similarly, the cash flow coefficient cannot be interpreted as the amount of dissaving per dollar of cash flow. Instead, it represents the amount of dissaving for a firm that is given a extra dollar of cash flow, relative to a group of firms of similar size and with similar levels of Tobin’s $q$.

Our simulations provide further insight into the GMM results. Recall that in the regressions on simulated data, Tobin’s $q$ in part reflects the value of cash to the firm and therefore is likely to pick up the income effect. If one uses a noisy proxy for true Tobin’s $q$ and therefore controls only for part of its variation, cash flow ends up picking up this income effect. Although the model provides some intuition for the result, it does not provide a complete explanation, because it is difficult to believe that the income effect on saving is extremely strong. However, Tobin’s $q$ picks up not only the income effect but also other reasons for saving, such as changes in agency issues, that are capitalized by the stock market. Clearly, not controlling for these motivations for saving causes cash flow to pick them up and contributes to making its OLS coefficient positive.

We now turn to the reliability of these regression results. Table III presents four summary statistics from the yearly regressions that underlie the Fama-MacBeth estimates in Table II: the fraction of years in which the cash flow coefficients are negative, in which the cash flow coefficients are significantly negative at the 5% level, in which the model overidentifying restrictions are rejected
at the 5% level, and in which the null of no model identification is rejected at the 5% level. In nearly all years for all countries we find negative cash flow coefficients, most of which are significant for the United States. Fewer are significant in the other countries. Sample size affects our ability to find statistical significance when using the GMM estimator because a great deal of data are required to estimate high-order moments with precision. For example, Canada, France, and Germany have small sample sizes. Another difficulty leading to insignificant coefficients is our frequent failure to reject the null of an unidentified model in Canada, France, and Germany. This result occurs in part because Tobin’s \( q \) has a less skewed distribution in these countries than it does in the United States. Both of these problems manifest themselves in high standard errors. Given these difficulties, our finding of a high incidence of significant Fama-MacBeth coefficients is all the more striking.

[Table III about here.]

Finally, Table III shows that we fail to reject the overidentifying restrictions from our yearly GMM estimates in all countries for most years. This result mitigates concerns about possible model misspecification. For example, although \( u_i \) is by construction uncorrelated with \((\chi_i, w_i)\), it may not be independent of \((\chi_i, w_i)\). Similarly, \( \varepsilon_i \) may not be independent of \((\chi_i, w_i)\), the true form of the regression (10) may be nonlinear, or the sample may not be i.i.d. Nonetheless, even though the classical errors-in-variables model is not a perfect representation of the relationship between saving, cash flow, and Tobin’s \( q \), our tests indicate that we have a useful approximation. Our failure to reject is even more interesting in light of the Monte Carlo result in the Appendix that this test tends to overreject slightly in finite samples. More important, this Appendix shows that this test has good finite-sample power to detect modest amounts of model misspecification.

Next we test our second prediction that the cash flow coefficient is more negative for firms with more costly external finance. In contrast to the case of our simulations, in which we know exactly which firms are constrained, no perfect measures of the severity of external finance constraints exist. Nonetheless, we use two commonly used measures: firm size and the existence of a bond rating.\(^{10}\) Although our Japanese sample is large enough to perform sample splits, we omit these results for brevity because they are similar to, but slightly weaker than, those obtained using U.S. data.
We define a firm as large if the book value of its assets lies above the 67th percentile and small if its assets lie below the 33rd percentile. In the literature on finance constraints, size is often used as an indicator of the cost of raising external funds, and Hennessy and Whited (2007) demonstrate that it is an accurate indicator. Firm size confers an important advantage over other indicators such as dividend payout. Size can be considered exogenous, because it is not a choice variable for the manager in the short run and because our estimates exploit cross-sectional data variation instead of time series variation. The intuition behind using the existence of a bond rating is that a firm with a bond rating has undergone a great deal of public scrutiny and is less likely to encounter the asymmetric information problems that lead to finance constraints. We regard bond ratings as exogenous, because agencies that provide bond ratings tend to base their judgments more on a consistent history of good financial and operating performance than on current operating decisions.

The first half of Table IV presents the results from these two sample splits. Our OLS results confirm those in Almeida, Campello, and Weisbach (2004) that small firms and firms without bond ratings have a stronger response of saving to cash flow than their unconstrained counterparts. In contrast, all of our GMM estimates are negative, and the estimates for the large firms are significantly more negative than those for the small firms. This result is similar to that in Almeida, Campello, and Weisbach (2004) inasmuch as the coefficients for the unconstrained firms exceed those for the constrained firms. Our coefficient estimates are simply shifted down from theirs, although the unconstrained-firm coefficients decrease more. This pattern makes sense because measurement error in Tobin’s $q$ biases the cash flow coefficient upward, because the amount of the measurement error bias depends on the correlation between Tobin’s $q$ and cash flow, and because these two variables are more highly correlated in the unconstrained groups than in the constrained groups. However, this result does not support the idea in Almeida, Campello, and Weisbach (2004) that unconstrained firms should have no sensitivity of saving to cash flow; nor does it support our model’s prediction that firms with more costly external finance have more negative cash flow coefficients. We conjecture that the effect of uncertainty on saving dwarfs the effect of the cost of external finance, which should be present in light of the evidence in Hennessy and Whited (2007) that both small and large firms face costly external finance and that the cost is larger for small
firms. When we examine differences between the large and small firms, one characteristic that stands out is the marked difference in the degree of uncertainty that they face. We estimate a first-order panel autoregression of the ratio of operating income to assets for both groups of firms, using the technique in Holtz-Eakin, Newey, and Rosen (1988). The standard deviations of the error term for the small firms and the firms without bond ratings are 0.149 and 0.107, respectively. In contrast, the standard deviations for the large firms and the firms with bond ratings are only half as large at 0.075 and 0.067. As demonstrated in the model simulations, firms that face a great deal of uncertainty do not make large changes in their cash holdings in response to income shocks.

[Table IV about here.]

It is natural to examine at this point the conjecture that uncertainty matters more than finance constraints by testing our third and fourth predictions, which concern the relation between saving propensities and the serial correlation and variance of income. To this end we estimate an $AR(1)$ for operating income (scaled by total assets) firm-by-firm, only using firms with at least six consecutive observations. We then sort our sample by the estimates of serial correlation and residual standard deviation, throw out the middle third, and compare saving propensities across the top and bottom thirds. Because firm-by-firm autoregressions produce noisy estimates, we discard the middle third to minimize the possibility that we incorrectly classify individual firms.11

The next four lines of Table IV contain the results for firms grouped by the standard deviation of the residual of the $AR(1)$ process for operating income. As predicted by the model, the low standard deviation group has a large, significantly negative cash flow coefficient, whereas the high standard deviation group has a cash flow coefficient that is statistically different both from zero and from the coefficient in the high standard deviation group. Not surprisingly, given the results on large and small U.S. firms, the low standard deviation group contains firms considerably larger than those in the high standard deviation group. The mean level of assets for the former is 4,053 million 1997 dollars, and the mean level of assets for the latter is 63 million 1997 dollars. This result is of further interest because it demonstrates that the degree of uncertainty is not proxying
for finance constraints. Otherwise we should have seen a more negative cash flow coefficient in the high uncertainty firms, but we do not.

The results from examining samples with low and high serial correlation are in the final four lines of Table IV. The GMM estimates of the cash flow coefficient are significantly higher for the high serial correlation firms than for the low serial correlation firms. In the first group the median serial correlation is near 0.8, and in the second it is near zero. For the low serial correlation group the cash flow coefficient is only significant if one considers the asymptotic 5% critical value instead of the bootstrapped 5% critical value. These results support the simulation that shows a low saving sensitivity if the serial correlation of the income process is low, but a large negative saving sensitivity if the serial correlation is high.

Just as we do for our full-sample results, we also present in Table V summary statistics describing the yearly regressions underlying the Fama-MacBeth estimates. We once again find a very low incidence of overidentifying restriction rejections and a fairly high incidence of identification test rejections. Both of these pieces of evidence support the reliability of our split-sample results.

[Table V about here.]

We next test directly whether uncertainty matters as much as finance constraints by running a regression of saving on Tobin’s $q$, the log of assets, cash flow, a constraint dummy, a low uncertainty dummy, the interaction of each of these dummies with cash flow, the interaction of the two dummies with each other, and the triple interaction of both of these dummies with cash flow. We use each of our measures of finance constraints separately. The results are in Table VI, which for brevity reports only the cash flow coefficient and the coefficients on the interaction terms. For both size and the existence of a bond rating, we find a negative coefficient on cash flow and a positive coefficient on the interaction of cash flow and the constraint dummy. These coefficient estimates confirm the differential sensitivity results in Table IV.

[Table VI about here.]
Two pieces of evidence are of particular interest in Table VI. Both concern the figure in the last column, which is the sum of the coefficients on the three interaction terms. It measures the net effect of being constrained and having low uncertainty. When we use size to proxy for finance constraints, this coefficient is positive, but insignificantly different from zero if we use our bootstrapped critical values. When we use the bond-rating dummy, this coefficient is negative and significant. In the first case stripping away the high uncertainty firms from the constrained group leaves no differential sensitivity between this smaller constrained group and the rest of the sample. In the second case stripping away the high uncertainty firms allows the predicted negative relation between saving propensities and finance constraints to be apparent. Nonetheless, the main message of this table is that both uncertainty and finance constraints affect the cash flow coefficient and that whatever its sign, this coefficient cannot be used as a summary measure of finance constraints.

Although we demonstrate in the Appendix the accuracy and good finite sample properties of the estimators we use, it would be reassuring to find a different measurement-error consistent estimator that produced similar results. To this end, we construct the proxy for Tobin’s \( q \) in Cummins, Hassett, and Oliner (2006) and Bond and Cummins (2001), which is built on the idea that Tobin’s \( q \) should be the expected discounted profits from using capital. This measure uses analysts’ estimates of earnings and growth rates from I/B/E/S to proxy for expected future profits. Although it is difficult to argue that this proxy is superior or inferior to the usual market-based proxy, these papers do argue convincingly that one can use lagged observable variables as instruments, because these proxies are based on forecasts that are, by definition, orthogonal to information known at the time of the forecast. As argued in Erickson and Whited (2000), such is not the case for the usual market-based proxy for Tobin’s \( q \) because the measurement errors in this proxy are highly serially correlated. When we use the proxy based on analysts’ forecasts, and use the same instrument sets as in Cummins, Hassett, and Oliner (2006) and Bond and Cummins (2001), we replicate their result that the coefficient on cash flow in an investment regression is insignificantly different from zero. Using this proxy in a saving regression produces a coefficient of -0.310 with a heteroskedasticity and autocorrelation robust standard error of 0.057. Although this result confirms the rest of our evidence, we use this technique only as a robustness check because the data required to construct
this alternate proxy for Tobin’s $q$ are available for less than half of our original sample.\textsuperscript{12}

It is interesting to compare our results with those in Erickson and Whited (2000), who find cash flow coefficients near zero when applying measurement-error consistent estimators to a regression of investment on $q$ and cash flow. This result may seem puzzling because the investment and saving regressions have the same right-hand-side variables, because the magnitudes of the OLS estimates of the cash flow coefficients are similar in the two regressions, and because measurement error bias propagates through the covariance matrix of the regressors. The difference between our results and theirs lies in their use of a different proxy for investment opportunities, whose estimate of $\tau^2$ is about 0.4—twice as large as the estimate for our proxy. Therefore, even though both studies find positive OLS estimates of cash flow coefficients, in the saving regression these estimates are more severely biased upward than they are in the investment regression. This differential bias implies that the true coefficient in the saving regression is negative and the true coefficient in the investment regression is zero. Not surprisingly, in results not reported for brevity, we are unable to find positive OLS cash flow coefficients in our saving regressions when using the proxy from Erickson and Whited (2000); further, as is the case here, we find negative GMM estimates of the cash flow coefficients.

We next investigate the two interesting instances in which our model predicts a positive coefficient on cash flow: cash flow shocks independent of productivity and intermittent investment. A careful examination of the first situation is beyond the scope of this paper inasmuch as empirical examples of exogenous movements in cash flow are very difficult to find. Indeed, we are only aware of three such studies in the empirical investment literature: Blanchard, Lopez-de-Silanes, and Schleifer (1994), Lamont (1997), and Rauh (2006). Nonetheless, we examine this issue in an informal way by looking at the two industries in which Tobin’s $q$ and cash flow are negatively correlated: SIC 28 (chemicals) and SIC 38 (measurement instruments). These two industries account for approximately 10\% of our observations. Our measurement-error consistent estimators produce mixed results. In approximately half of the years in our sample we find positive coefficients and some of them are significantly different from zero. Although none of the negative coefficients are significant, the average of the coefficients over the sample period is negative. Although this result
tells us nothing about a causal relationship between the importance of exogenous cash-flow shocks and the sign of the cash flow effect, it is interesting in that it is consistent with the prediction of our model that profit shocks unrelated to productivity can produce a positive sensitivity. To understand the source of our result, we examine the dimensions on which these firms differ from the rest of the sample. They are remarkably similar in terms of size, the autocorrelation and variance of profits, the incidence of bond ratings, and leverage. They do invest less and hold more cash than the rest of the sample, but these differences are small. The similarity between the two subsamples makes it difficult to attribute the positive cash flow coefficients to the presence of finance constraints. Instead, we conjecture that technological or competitive factors are driving our result.

To examine the second situation we look at firms in the lowest size decile in our sample. As shown in Whited (2006), these micro firms are the only ones in Compustat whose investment is intermittent. Indeed, in our sample, these firms have an investment rate less than 1% in 12% of the firm-year observations. The corresponding figure for the rest of the sample is 4%. In accordance with the prediction of our model that intermittent investment produces a positive sensitivity of saving to cash flow, we do find a positive and significant saving sensitivity in most years. However, as above, it is difficult to attribute this result to finance constraints because these firms also have highly variable income with low serial correlation. It is also worth noting that we obtain negative sensitivities in the other nine size deciles. In sum, although we can isolate some samples for which the sign of the cash flow coefficient is positive, the bulk of our results point to a negative coefficient. This result should not be surprising in light of the evidence in Smets and Wouters (2007) and Whited (2006) that the conditions required for a positive sensitivity are unlikely to be pervasive.

V. Conclusion

The issue of corporate saving has recently received much attention, in large part because of the build-up of liquid assets in recent years by both U.S. and European firms. Prior empirical research, including papers by Opler, Pinkowitz, Stulz, and Williamson. (1999), Faulkender and Wang (2006), Almeida, Campello, and Weisbach (2004), and Khurana, Martin, and Pereira. (2006), has addressed
two related issues: why firms hold cash and why firms save, that is, change their cash holdings. For the most part, we address the second issue. In so doing, we take care to model a firm’s saving, financing, and real investment decisions simultaneously in a stochastic, dynamic framework, to form a strong link between our theory and our empirical tests, and to account for measurement error in our empirical work.

This approach leads us to conclusions richer than previous theoretical and empirical results. Our dynamic model predicts that, controlling for Tobin’s \( q \), the firm counteracts movements in cash flow with opposite movements in saving. This negative propensity to save occurs because a positive productivity shock causes both cash flow and the marginal product of capital increase. A substitution effect then induces the firm to use some of its cash stock to buy capital goods that have become relatively more productive, that is, to dissave and invest. In contrast, the sensitivity of saving to cash flow in earlier models with one-period saving decisions, such as the one in Almeida, Campello, and Weisbach (2004), is typically positive because good cash flow news is modeled as an increase in the firm’s endowment. The marginal product of capital is unaffected, and the firm has no incentive to transform liquid assets into productive assets. Instead, the firm wants to hold high levels of cash to avoid costly external finance. This income effect operates in our model as well, but our model extends this earlier work by allowing for the existence of a strong substitution effect, which is in turn derived from an endogenous investment decision and variation in capital productivity.

We find strong empirical support for our negative sensitivity result in data from six countries. When we estimate this sensitivity using OLS, we find the standard result in the literature that the sensitivity of saving to cash flow is positive. However, when we correct econometrically for measurement error in Tobin’s \( q \), we find the opposite result, except in two small subsamples of firms. We also find that the saving sensitivity increases in absolute value with the serial correlation of income and decreases with the variance of income shocks. Interestingly, the effect of uncertainty on the propensity to save out of cash flow is empirically at least as strong as the effect of finance constraints. Consequently, propensities to save cannot be used as summary measures of the cost of external finance or of any of the other multitude of real and financial factors that affect these
propensities.

Taken together, our model and evidence provide insight into the process of corporate saving. In general, we describe how real investment decisions are critical for understanding cash accumulation. We also demonstrate the ways in which dynamic models can clarify the economic forces that drive reduced-form regressions. Finally, our results reemphasize that any model estimation involving Tobin’s $q$ can suffer similarly from significant measurement error bias.
REFERENCES


Geary, R. C., 1942, Inherent relations between random variables, *Proceedings of the Royal Irish Academy A* 47, 63-76.


Footnotes

1. Our work is also related to empirical work on the level (instead of the accumulation) of corporate cash, surveyed in Faulkender and Wang (2006) and Foley, Hartzell, Titman, and Twite (2007).

2. The Appendix can be found at http://www.afajof.org/supplements.asp.

3. Replacing $ck$ with a fixed number, $F$, changes the analysis little because the capital stock is bounded.

4. Deflating the variables in (9) by $(k + p)$ instead of by $k$ changes the results little.

5. Simulations in which $\rho$ increases but the variance of the process is held constant produces a similar result, except that the rise in cash holdings for very low $\rho$ flattens out.

6. Quadratic physical adjustment costs do not cause this result because it remains after turning off these costs.

7. In a similar setting Almeida, Campello, and Weisbach (2007) allow for either a positive or negative effect of internal resources on cash. The difference lies in the introduction of multiple productive assets with varying liquidity.

8. Tables containing the GMM results using fixed effects are in the online Appendix.

9. Our OLS estimates of the coefficients on $q$ and cash flow are larger than those in Almeida, Campello, and Weisbach (2004) because we use trimmed data and they do not. When we do not trim out data, we get OLS results that are similar to theirs.

10. We omit other commonly used measures of finance constraints such as dividend payout and the KZ index (from Kaplan and Zingales (1997)) because they are endogenously determined with investment. We do not consider commercial paper ratings because too few firms have them.

11. We also estimate these autoregressions on an industry level with almost identical results.
12. Almeida, Campello, and Weisbach (2004) use analysts’ earnings estimates as instruments for Tobin’s $q$ and find a positive, significant cash flow coefficient. When we do this we find a positive insignificant coefficient. We attribute the insignificance to poor instrument quality, that is, the low correlation between analysts’ earnings estimates and Tobin’s $q$.

13. We also examine firms in highly concentrated industries, firms in competitive industries, firms with low and high return volatility, and firms in industries with low and high depreciation rates. These sample splits capture, respectively, curvature of the profit function (which is a function of demand elasticity), uncertainty about both productivity and cost shocks, and depreciation rates. None of these subsamples produce a positive cash flow coefficient.
Table I
Summary Statistics

Calculations are based on a sample of U.S. nonfinancial firms from Compustat from 1972 to 2006 and a sample of international firms from Global Vantage from 1994 to 2005. Investment, cash flow, the cash stock, and the change in the cash stock are all deflated by total assets. Total assets are expressed in millions of 1997 U.S. dollars.

<table>
<thead>
<tr>
<th>Country</th>
<th>Investment</th>
<th>Market to Cash Flow</th>
<th>Cash Flow</th>
<th>Change in Cash Stock</th>
<th>Cash Stock</th>
<th>Total Assets</th>
<th>Average Obs. per Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>Mean 0.0967</td>
<td>1.4311</td>
<td>0.1341</td>
<td>0.0031</td>
<td>0.1119</td>
<td>1,522</td>
<td>2,258</td>
</tr>
<tr>
<td></td>
<td>Median 0.0621</td>
<td>1.1831</td>
<td>0.1405</td>
<td>0.0003</td>
<td>0.0587</td>
<td>119</td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>Mean 0.1212</td>
<td>1.3345</td>
<td>0.0772</td>
<td>0.0042</td>
<td>0.0919</td>
<td>1,037</td>
<td>341</td>
</tr>
<tr>
<td></td>
<td>Median 0.0694</td>
<td>1.1312</td>
<td>0.0874</td>
<td>0.0000</td>
<td>0.0295</td>
<td>247</td>
<td></td>
</tr>
<tr>
<td>United Kingdom</td>
<td>Mean 0.0755</td>
<td>1.5043</td>
<td>0.0809</td>
<td>0.0025</td>
<td>0.1174</td>
<td>1,054</td>
<td>796</td>
</tr>
<tr>
<td></td>
<td>Median 0.0442</td>
<td>1.3134</td>
<td>0.0968</td>
<td>0.0001</td>
<td>0.0710</td>
<td>115</td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>Mean 0.0282</td>
<td>1.1940</td>
<td>0.0326</td>
<td>-0.0027</td>
<td>0.1647</td>
<td>461</td>
<td>2,070</td>
</tr>
<tr>
<td></td>
<td>Median 0.0182</td>
<td>1.0978</td>
<td>0.0307</td>
<td>-0.0021</td>
<td>0.1389</td>
<td>230</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>Mean 0.0107</td>
<td>1.2511</td>
<td>0.0615</td>
<td>-0.0078</td>
<td>0.1279</td>
<td>1,116</td>
<td>353</td>
</tr>
<tr>
<td></td>
<td>Median 0.0072</td>
<td>1.1102</td>
<td>0.0686</td>
<td>-0.0010</td>
<td>0.0913</td>
<td>84</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>Mean 0.0519</td>
<td>1.2489</td>
<td>0.0704</td>
<td>-0.0102</td>
<td>0.1075</td>
<td>1,418</td>
<td>332</td>
</tr>
<tr>
<td></td>
<td>Median 0.0384</td>
<td>1.1498</td>
<td>0.0793</td>
<td>-0.0023</td>
<td>0.0586</td>
<td>122</td>
<td></td>
</tr>
</tbody>
</table>
Calculations are based on a sample of U.S. nonfinancial firms from Compustat from 1972 to 2006 and a sample of international firms from Global Vantage from 1994 to 2005. GMM estimates are from the fourth-order estimator in Erickson and Whited (2000). The dependent variable is the change in the stock of cash divided by total assets. $CF$ stands for cash flow divided by total assets; $q$ stands for the market-to-book ratio; and $\tau^2$ is an index of measurement quality for the market-to-book ratio that varies between zero and one. Fama-MacBeth standard errors are below the estimates in parentheses. An asterisk indicates that the $t$-statistic exceeds the 5% bootstrapped critical value. A dagger indicates that the $t$-statistic exceeds the 5% asymptotic critical value.

<table>
<thead>
<tr>
<th>Country</th>
<th>OLS</th>
<th></th>
<th></th>
<th>GMM4</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$q$</td>
<td>$CF$</td>
<td>$R^2$</td>
<td>$q$</td>
<td>$CF$</td>
<td>$R^2$</td>
</tr>
<tr>
<td>United States</td>
<td>0.029*†</td>
<td>0.103*†</td>
<td>0.112*†</td>
<td>0.283*†</td>
<td>-0.397*†</td>
<td>0.440*†</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.009)</td>
<td>(0.010)</td>
<td>(0.016)</td>
<td>(0.060)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>Canada</td>
<td>0.045*†</td>
<td>0.053*†</td>
<td>0.144*†</td>
<td>0.213*†</td>
<td>-0.076*†</td>
<td>0.495*†</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.025)</td>
<td>(0.026)</td>
<td>(0.018)</td>
<td>(0.022)</td>
<td>(0.051)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.009†</td>
<td>0.103*†</td>
<td>0.047*†</td>
<td>0.427</td>
<td>-0.485*†</td>
<td>0.356*†</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.016)</td>
<td>(0.013)</td>
<td>(0.076)</td>
<td>(0.168)</td>
<td>(0.042)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.019*†</td>
<td>0.141*†</td>
<td>0.049*†</td>
<td>0.318*†</td>
<td>-0.162*†</td>
<td>0.255*†</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.019)</td>
<td>(0.005)</td>
<td>(0.040)</td>
<td>(0.037)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>France</td>
<td>0.021*†</td>
<td>0.126*†</td>
<td>0.084*†</td>
<td>0.263*†</td>
<td>-0.304*†</td>
<td>0.303*†</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.033)</td>
<td>(0.013)</td>
<td>(0.084)</td>
<td>(0.097)</td>
<td>(0.060)</td>
</tr>
<tr>
<td>Germany</td>
<td>0.018†</td>
<td>0.078*†</td>
<td>0.082*†</td>
<td>0.310*†</td>
<td>-0.200*†</td>
<td>0.354*†</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.020)</td>
<td>(0.018)</td>
<td>(0.073)</td>
<td>(0.087)</td>
<td>(0.069)</td>
</tr>
</tbody>
</table>
Calculations are based on a sample of U.S. firms from Compustat from 1972 to 2006 and a sample of international firms from Global Vantage from 1994 to 2005. The first column contains the fraction of the yearly estimates of the cash flow coefficient that are negative. The second column contains the fraction of the yearly estimates of the cash flow coefficient that are significantly negative at the 5% level, using asymptotic critical values. The third column contains the fraction of the yearly tests of overidentifying restrictions that produce rejections at the 5% level. The fourth column contains the fraction of the yearly identification tests that produce rejections at the 5% level.

<table>
<thead>
<tr>
<th>Country</th>
<th>Fraction of Negative Cash Flow Coefficients</th>
<th>Fraction of Significant Negative Cash Flow Coefficients</th>
<th>Fraction of Overidentifying Restriction Rejections</th>
<th>Fraction of Identification Test Rejections</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
<td>0.886</td>
<td>0.714</td>
<td>0.085</td>
<td>0.857</td>
</tr>
<tr>
<td>Canada</td>
<td>0.833</td>
<td>0.250</td>
<td>0.083</td>
<td>0.417</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.833</td>
<td>0.333</td>
<td>0.250</td>
<td>0.917</td>
</tr>
<tr>
<td>Japan</td>
<td>1.000</td>
<td>0.333</td>
<td>0.250</td>
<td>0.917</td>
</tr>
<tr>
<td>France</td>
<td>0.750</td>
<td>0.333</td>
<td>0.083</td>
<td>0.333</td>
</tr>
<tr>
<td>Germany</td>
<td>0.667</td>
<td>0.250</td>
<td>0.000</td>
<td>0.500</td>
</tr>
</tbody>
</table>
Calculations are based on a sample of U.S. nonfinancial firms from Compustat from 1972 to 2006. GMM estimates are from the fourth-order estimator in Erickson and Whited (2000). The dependent variable is the change in the stock of cash divided by total assets. \( CF \) stands for cash flow divided by total assets; \( q \) stands for the market-to-book ratio; and \( \tau^2 \) is an index of measurement quality for the market-to-book ratio that varies between zero and one. Serial correlation is the first-order autoregressive coefficient on the ratio of operating income to assets, and standard deviation is the standard deviation of the residual from this regression. Fama-MacBeth standard errors are below the estimates in parentheses. An asterisk indicates that the \( t \)-statistic exceeds the 5% bootstrapped critical value. A dagger indicates that the \( t \)-statistic exceeds the 5% asymptotic critical value.

### Table IV

**Split Sample Regressions: United States**

<table>
<thead>
<tr>
<th>Subsample</th>
<th>OLS</th>
<th>GMM4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( q )</td>
<td>( CF )</td>
</tr>
<tr>
<td>Small</td>
<td>0.045*†</td>
<td>0.134*†</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Large</td>
<td>0.006*†</td>
<td>0.083*†</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>No Bond Rating</td>
<td>0.032*†</td>
<td>0.110*†</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Bond Rating</td>
<td>0.016*†</td>
<td>0.046†</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>High Standard Deviation</td>
<td>0.037*†</td>
<td>0.128*†</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Low Standard Deviation</td>
<td>0.014*†</td>
<td>0.081†</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>High Serial Correlation</td>
<td>0.023*†</td>
<td>0.088*†</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Low Serial Correlation</td>
<td>0.033*†</td>
<td>0.122†</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.009)</td>
</tr>
</tbody>
</table>
Table V  
Yearly Saving Regressions Summary: Split Samples

Calculations are based on a sample of U.S. nonfinancial firms from Compustat from 1972 to 2006. The first column contains the fraction of the yearly estimates that are negative. The second column contains the fraction of the yearly estimates of the cash flow coefficient that are significantly negative at the 5% level, using asymptotic critical values. The third column contains the fraction of the yearly tests of overidentifying restrictions that produce rejections at the 5% level. The fourth column contains the fraction of the yearly identification tests that produce rejections at the 5% level.

<table>
<thead>
<tr>
<th></th>
<th>Fraction of Negative Cash Flow Coefficients</th>
<th>Fraction of Significant Negative Cash Flow Coefficients</th>
<th>Fraction of Overidentifying Restriction Rejections</th>
<th>Fraction of Identification Test Rejections</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>0.571</td>
<td>0.371</td>
<td>0.057</td>
<td>0.629</td>
</tr>
<tr>
<td>Large</td>
<td>0.886</td>
<td>0.429</td>
<td>0.171</td>
<td>0.686</td>
</tr>
<tr>
<td>No Bond Rating</td>
<td>0.742</td>
<td>0.600</td>
<td>0.114</td>
<td>0.857</td>
</tr>
<tr>
<td>Bond Rating</td>
<td>0.857</td>
<td>0.600</td>
<td>0.029</td>
<td>0.571</td>
</tr>
<tr>
<td>High Standard Deviation</td>
<td>0.686</td>
<td>0.514</td>
<td>0.086</td>
<td>0.686</td>
</tr>
<tr>
<td>Low Standard Deviation</td>
<td>0.800</td>
<td>0.600</td>
<td>0.200</td>
<td>0.886</td>
</tr>
<tr>
<td>High Serial Correlation</td>
<td>0.971</td>
<td>0.657</td>
<td>0.086</td>
<td>0.714</td>
</tr>
<tr>
<td>Low Serial Correlation</td>
<td>0.571</td>
<td>0.257</td>
<td>0.171</td>
<td>0.657</td>
</tr>
</tbody>
</table>
Table VI
Uncertainty versus Finance Constraints

Calculations are based on a sample of U.S. nonfinancial firms from Compustat from 1972 to 2006 GMM estimates are from the fourth-order estimator in Erickson and Whited (2000). The dependent variable is the change in the stock of cash divided by total assets. CF stands for cash flow divided by total assets; DC is a dummy variable that takes a value of one if a firm is classified as constrained, and zero otherwise. DL is a dummy variable that takes a value of one if a firm is classified as having low uncertainty, and zero otherwise. The regression specification also includes the market-to-book ratio and the log of book assets. “Sum” refers to the sum of the coefficients in columns 3 to 5. Size and bond rating are the two financial constraint indicators. Fama-MacBeth standard errors are below the estimates in parentheses. An asterisk indicates that the t-statistic exceeds the 5% bootstrapped critical value. A dagger indicates that the t-statistic exceeds the 5% asymptotic critical value.

<table>
<thead>
<tr>
<th>Constraint Indicator</th>
<th>CF</th>
<th>CF × DC</th>
<th>CF × DL</th>
<th>CF × DC × DL</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Size</td>
<td>-0.384†</td>
<td>0.347†</td>
<td>-0.653†</td>
<td>0.443†</td>
<td>0.163†</td>
</tr>
<tr>
<td></td>
<td>(0.070)</td>
<td>(0.076)</td>
<td>(0.082)</td>
<td>(0.075)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>Bond Rating</td>
<td>-0.533†</td>
<td>0.318†</td>
<td>-0.972†</td>
<td>0.478†</td>
<td>-0.177†</td>
</tr>
<tr>
<td></td>
<td>(0.070)</td>
<td>(0.062)</td>
<td>(0.097)</td>
<td>(0.087)</td>
<td>(0.052)</td>
</tr>
</tbody>
</table>
Figure 1. Policy functions. This figure depicts the optimal response of investment, saving, cash flow, and distributions/equity issuance in response to the productivity shock, $z$, in the revenue function $zk^\theta$. The first panel depicts the response of the smallest firm in the simulated sample, the second panel depicts the response of the median firm in the simulated sample, and the third depicts the response of the largest firm in the simulated sample.
Figure 2. The ratio of cash to assets. This figure depicts the relation between various model parameters and the average optimal ratio of the stock of cash to assets. Each graph is constructed by varying the model parameter on the horizontal axis. For each of these parameter values the model is solved and simulated, and the average ratio of cash to assets is taken over all of the simulated data points.
Figure 3. The cash flow sensitivity of cash. This figure depicts the relation between various model parameters and the sensitivity of the change in the cash stock to cash flow, holding constant Tobin’s q and the capital stock. Each graph is constructed by varying the model parameter on the horizontal axis. For each of these parameter values the model is solved and simulated, and the average sensitivity is calculated over all of the simulated data points.