Testing Q theory with financing frictions

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Received 18 August 2003; received in revised format 26 July 2005; accepted 14 December 2005

Abstract

We develop a Q theory of investment under financing constraints. The firm invests and saves optimally facing convex costs of external equity, overhang from outstanding debt, and collateral constraints on new borrowing. Overhang and costs of external equity discourage investment. Conversely, firms anticipating collateral constraints experience a side benefit from investing as installed capital relaxes future constraints. Empirical tests support the model. Conditional on average Q, investment is lower for equity issuers and for firms with large debt overhang. The Kaplan and Zingales and the Whited and Wu indices are used as proxies for future collateral constraints. Consistent with the model, both indices enter investment regressions positively.

JEL classification: G31; E22.
Keywords: Q theory; Investment; Financial frictions

We would like to thank an anonymous referee, John Leahy, and seminar participants at the University of California, Berkeley, and the University of Wisconsin, Madison, for helpful comments.

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1. Introduction

The effect of financing imperfections on investment remains a topic of continued interest.[1] The workhorse structural model in this literature is the Q theory developed by Hayashi (1982), who shows that marginal q is a sufficient statistic for investment when there are convex costs of adjusting the capital stock. Of greater interest to empiricists is Hayashi's proof that average Q, which is observable, is equal to the inherently unobservable marginal q when profits are linear in capital and financing is frictionless. Elegance aside, the utility of Hayashi's model in empirical corporate finance is limited, because it provides no guidance regarding structural tests for financial market imperfections. For example, within the Q theory, what kind of investment behavior distinguishes credit rationing from, say, lemons premia in equity markets?

Few workable alternatives to the Hayashi framework have been offered. The objective of this paper is to formulate and test a Q theory of investment in the presence of financial frictions. The model of Hayashi (1982) is extended to allow for uncertainty, a cash buffer-stock, collateral constraints on intermediated bank loans, debt overhang arising from long-term public debt, and convex costs of external equity. Some recent papers simulate dynamic structural models in which the firm faces a similar set of frictions.[2] The contribution of our model is twofold. First, in contrast to simulation-based studies, we derive an easy-to-estimate optimality condition in which investment is linear in the relevant variables. Second, we derive a relation between marginal and average Q in a setting in which the firm finances optimally in the presence of a rich set of frictions.

An attractive feature of the model is that the empirical investment equation contains terms that can be linked directly to each of the three distortions: convex costs of external equity, collateral constraints, and debt overhang. The empirical section of the paper uses US firm-level data to test three predictions related to the three frictions. The first prediction of the model is that, conditionally, investment is decreasing in an interaction variable: average Q multiplied by capital-normalized equity issuance. This prediction could run counter to casual intuition. After

[1] See Stein (2003) for a recent survey.

[2] See Gomes (2001), Cooper and Ejarque (2003), and Hennessy and Whited (2006).

all, firms tend to issue equity to finance large investments. However, average Q already captures shocks to the investment opportunity set. The interaction variable captures the following effect. Suppose firms A and B have the same investment opportunity set, but firm B has no cash on hand and must float equity to fund its investment program. Convex costs of external equity raise the marginal cost of each unit of capital acquired by firm B. Consequently, firm B invests less than A. Consistent with the model, estimated coefficients on this interaction variable are always negative and typically statistically significant. Further, correcting for measurement error in observed Q increases the absolute magnitude of the coefficients three- to fivefold. The measurement-error consistent estimates are also economically significant. For example, according to our estimates, the median equity-issuing firm in our sample would increase investment by approximately 5% if it were able to finance with internal funds.

The second prediction of the model could also run counter to intuition: ceteris paribus, firms anticipating binding collateral constraints on future borrowing invest more today. For example, consider a high Q firm anticipating valuable investment opportunities in the future. If this firm has a high ratio of loans to tangible capital, lenders are reluctant to provide funds for these future investments. For such a firm, installing capital today provides a spillover benefit, because this capital serves as collateral for future loans. This spillover benefit increases investment incentives at the margin.

We use the Kaplan and Zingales (hereafter, KZ, 1997) and Whited and Wu (hereafter, WW, 2006) indices to proxy for firms anticipating binding collateral constraints. The motivation is similar to that provided by Baker, Stein, and Wurgler (2003), who use the KZ index to identify equity dependent firms. First, both indices load in the right way onto firm characteristics suggestive of binding collateral constraints. For example, both indices are increasing in proxies for investment opportunities, increasing in financial leverage, and decreasing in internal funds. Second, both indices have been used in other contexts so concern over data mining should be assuaged. Consistent with the model, coefficients on the KZ and WW indices are always positive and significant. The signs of these coefficients are also robust to correcting for measurement error in observed Q.

The final friction tested is the debt-overhang effect. The overhang correction variable is the same as that derived by Hennessy (2004). The model presented in our paper is more general in that it includes endogenous cash retention, credit rationing, and costs of external equity. The theory predicts that overhang is most pronounced for firms with high leverage, high default probabilities, and high lender recoveries in default. Intuitively, the overhang correction is intended to pick up investment returns accruing to lenders as opposed to shareholders. Consistent with the theory, the overhang correction always enters negatively, is always statistically significant, and is robust to measurement error. Its magnitude is substantial. Our measurement-error consistent estimates indicate that the elasticity of investment with respect to leverage for the median firm is approximately -2. Because our model controls for other frictions that one might expect to be correlated with the overhang correction, these results suggest that the findings in Hennessy (2004) do not stem from omitted variables.

Hayashi (1985) and Chirinko (1987) pursue the same agenda undertaken in this paper, endogenizing finance in Q theoretic models of investment and then relating marginal q to average Q. These two models are more limited than the one here. The Hayashi (1985) model is essentially the trade-off theory, because the only frictions incorporated are taxes and reduced-form bankruptcy costs. The model of Chirinko (1987) does not allow for uncertainty. In addition, Chirinko's model loads financial market imperfections directly into the production function. For example, in his model, debt reduces current period profits. However, we show that agency costs of debt, such as the Myers (1977) overhang effect, are not isomorphic to foregone production today. Instead, debt overhang causes shareholders to ignore future investment returns accruing after default.

The distortions included in the model are motivated by a number of theoretical papers. Convex costs of external equity are intended to capture the effect of informational asymmetries. Myers and Majluf (1984) consider a firm with a single indivisible investment opportunity. They show that asymmetric information can raise the cost of external equity if the firm is pooled with those of lower quality. If the lemons problem is sufficiently severe, good firms find it optimal to pass up positive net present value (NPV) projects. Krasker (1986) presents a generalized model of adverse selection in equity markets. In his model, as in our model, the firm chooses the scale of new investment. Krasker shows that under rational expectations the shadow cost of external equity is convex. The reasoning is as follows. As the firm issues more equity, beliefs about the manager's private information are revised downward, lowering the price paid for both marginal and inframarginal shares.

Myers and Majluf (1984) argue that lemons premia associated with external equity create incentives to use retained earnings and debt as sources of funds. However, the lemons problem also limits the ability of firms to obtain debt financing on fair terms. Jaffee and Russell (1976) and Stiglitz and Weiss (1981) present models of the intermediated loan process, showing that banks could ration credit in response to adverse selection. These models are empirically relevant, because bank debt is the most important source of financing. For example, within a random sample of NYSE and Amex corporations, Houston and James (1996) find that the median percentage of bank debt in total debt is 77%.

Concerns regarding moral hazard on the part of the borrower may also lead to credit rationing. For example, Hart and Moore (1994) consider a setting in which the entrepreneur has the option to repudiate a debt contract. By threatening to withdraw valuable human capital from the project, the entrepreneur can extract debt forgiveness from the lender. Under any renegotiation-proof debt contract, the maximum loan balance at any time is capped at the liquidation value of physical assets. The Hart and Moore model seems to best fit bank debt, given its assumption of frictionless bargaining between the firm and lender.

Almeida and Campello (2004) devise a novel empirical test of theories emphasizing the importance of tangible capital in credit markets, e.g. Hart and Moore (1994), showing that exogenous cash shocks should have a larger effect on capital accumulation when the tangibility of capital is high. This is because a dollar windfall buys a unit of capital, which serves as collateral for another loan. When asset tangibility is high, the chain of collateralized investments ultimately results in a large multiplier effect. Almeida and Campello find that their prediction holds across a broad range of specifications. Our model and the Almeida and Campello model rely upon similar arguments. In both models, the firm and lender rationally account for the spillover effect provided by capital in terms of loosening the collateral constraint. However, the two papers rely upon different methods for empirical identification.

Myers (1977) shows that preexisting defaultable debt creates an overhang problem, with the problem being most severe for long-lived debt. In particular, an equity-maximizing manager underinvests relative to first-best whenever capital accumulation provides a positive spillover to existing lenders. Lang, Ofek, and Stulz (1996) present empirical evidence supportive of the Myers theory and consistent with our results. When book leverage is included as a regressor, along with average Q, leverage only enters significantly negative for low Q firms. This is precisely the set of firms for which the overhang correction is likely to be large.

Fazzari, Hubbard, and Petersen (1988) try to infer the magnitude of financing frictions based upon the sensitivity of investment to cash flow. Kaplan and Zingales (1997) question the formal justification for this traditional methodology. For example, in simulations of a dynamic model, Gomes (2001) shows that financial frictions are neither necessary nor sufficient for significant cash flow effects. In contrast to the traditional approach, the empirical tests in our paper do not rely upon cash flow coefficients and are derived explicitly from a dynamic structural model.

The Stein (2003) recent survey provides a convenient taxonomy. The model presented in this paper includes the main frictions discussed in his models of costly external finance: convex costs of external equity, credit rationing, and debt overhang. Excluded from our analysis are theories based upon agency conflicts between managers and shareholders. The omission reflects tractability concerns, not perceptions of relative importance.

Section 2 presents the model; Section 3, the data; and Section 4, the empirical estimation. Section 5 concludes. Proofs of all propositions are in Appendix A, a description of the data is in Appendix B, and the derivation of the generalized method of moments (GMM) estimators used in Section 4 in in Appendix C.

2. The model

The manager works in the interest of current shareholders. Investors are risk neutral and discount cash flows at the risk-free rate $r > 0$. The endogenous state variable K denotes the capital stock and I denotes investment. An exogenous state variable ε captures innovations in output prices, variable input costs, and productivity. Operating profits are

$$F(K, \varepsilon) - G(I, K). \tag{1}$$

The function F represents gross profits, excluding costs of installing new machinery or removing old machinery. The function G represents capital adjustment costs other than the direct price of capital. For example, a major investment program disrupts current business operations. The function G captures such effects. For simplicity, the price of capital is normalized to one.

Assumption 1 imposes standard restrictions on real technologies.

Assumption 1 *The gross profit function F is twice continuously differentiable, strictly increasing in both arguments, and homogeneous degree one in K. The adjustment cost function G is twice continuously differentiable, strictly convex, and homogeneous degree one in (I, K).*

The gross profit function (F) is linear in capital. For example, F is linear when the firm is a price-taker and the production function exhibits constant returns-to-scale. The implications of nonlinearity resulting from market power or decreasing returns or both are examined in the empirical tests. The costs of capital adjustment (G) are convex. Lucas (1967) argues that such adjustment costs are necessary to explain investment dynamics. Since then, convex adjustment costs have been standard in dynamic investment models.

There exists a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ supporting a Wiener process W_t. For each time $t \in [0, \infty)$, information revelation is described by the filtration $\mathcal{F}_t \subset \mathcal{F}$, where each $\mathcal{F}_t$ is the augmented σ-algebra generated by the Wiener process. The evolution of (K, ε) is described by

$$dK_t = (I_t - \delta K_t)dt \quad and \tag{2}$$

$$d\varepsilon_t = \mu(\varepsilon_t)dt + \sigma(\varepsilon_t)dW_t. \tag{3}$$

The parameter δ represents the depreciation rate. The drift and volatility for ε satisfy the necessary conditions for the existence of a unique solution to the stochastic differential equation as specified in Oksendal (2000). The diffusion process for the state variable ε is general enough to capture competitive dynamics affecting capital accumulation. For instance ε can be specified as an Ornstein-Uhlenbeck process to proxy for the effects of entry and exit.

Consistent with the theoretical literature on adverse selection in equity markets, we assume that the costs of external equity are convex. Anticipating, these costs cause the firm to retain funds to reduce reliance on external equity. If the firm lacks internal funds and debt capacity, convexity creates an incentive to spread equity flotations over time. In addition, we assume that smaller firms face higher marginal costs of external equity. This modeling approach is consistent with the findings of Altinkilic and Hansen (2000), who show that underwriting fees are increasing in the size of the issuance and decreasing in the size of the firm. In reality, numerous factors determine the cost of external equity for a particular firm. Our objective is to capture the key factors in a tractable manner. To this end, let X denote the dollar value of external equity finance. If $X < 0$, the firm is paying a dividend, not issuing new shares. The function $H(X, K)$ represents the cost of equity. Assumption 2 summarizes conditions imposed on H.

Assumption 2 *The cost of equity function H is twice continuously differentiable, strictly increasing and weakly convex in X, decreasing and convex in K, homogeneous degree one in (X, K), and satisfies $H(X, K) = X$ for all $X \leq 0$.*

In the model, dividends and equity flotations are both captured by the variable X. Negative values of X are equivalent to shareholders incurring a negative cost. That is, when $X < 0$, the firm is paying a dividend and shareholders receive X. When $X > 0$, the firm is raising external equity. The cost of external equity to current shareholders increases as the firm works its way up a convex cost schedule.

To capture the intermediated loan process, we assume the firm has access to a bank credit line. To account for this credit line, we add a second endogenous state variable B denoting the credit line balance. This credit line plays an important role, allowing the firm to avoid costly spikes in equity issuance when the investment opportunity set (ε) increases.

Following Hart and Moore (1994), it is assumed that the firm holds all bargaining power in the event of renegotiation with the bank. We then impose a renegotiation-proofness constraint on the bank credit line. The liquidation value of each unit of capital is $L(\varepsilon) \in (0, 1)$. We let L depend on the state of the industry (ε), motivated by Shleifer and Vishny (1992). Consistent with the empirical evidence presented by Carey (1995), the bank is assumed to be senior in the event of default. The bank imposes a collateral constraint ensuring $B_t \leq L(\varepsilon_t)K_t$. If $B_t > L(\varepsilon_t)K_t$, the credit line is not renegotiation proof, because the firm would make a take-it-or-leave-it offer pushing the bank down to its reservation value, which is $L(\varepsilon_t)K_t$. This particular functional form for the collateral constraint is also employed by Kiyotaki and Moore (1997) and Almeida and Campello (2004). In the event of default, absolute priority is obeyed. Assuming perfect competition in the intermediated loan market, the interest rate on the bank credit line must be r, because the collateral constraint ensures that the bank loan is risk-free.

In the presence of financing frictions, firms with free cash flow retain some portion of the funds to build up a cash buffer-stock. To accommodate such policies, the state variable B is allowed to go negative. There must be some cost to holding cash, otherwise no firm would ever distribute funds to shareholders. For example, Graham (2000) finds that cash retentions are tax-disadvantaged, because corporate tax rates generally exceed tax rates on interest income for the marginal bondholder. To account for such considerations, the firm earns an interest rate $r_s < r$ when it saves. For the sake of brevity, define $\widetilde{r}$ as

$$B \geq 0 \Rightarrow \widetilde{r} = r \quad and \tag{4}$$

$$B < 0 \Rightarrow \widetilde{r} = r_s < r. \tag{5}$$

The variable g denotes the financial control policy, regulating the growth of the bank credit line. The evolution of the credit line is determined by

$$dB_t = g_t dt. \tag{6}$$

The collateral constraint is enforced by restricting the instantaneous control g as

$$g \leq \overline{g}(K, B, \varepsilon) \equiv \gamma * [L(\varepsilon)K - B]. \tag{7}$$

The control constraint in Eq. (7) allows the firm to close a portion of the gap between the current credit line balance (B) and its upper bound (LK). The free parameter $\gamma > 0$ specifies the rate at which the firm is permitted to close the gap. This parameter can be set arbitrarily high, so that the firm can close the gap quickly. The chosen value of γ has no effect on our empirical investment equation.

The intuition for the credit line balance is as follows. When g_t and B_t are both positive, the firm is drawing on its credit line. When g_t is positive and B_t is negative, the firm is running down its buffer-stock of cash. When g_t is negative and B_t is positive, the firm is paying down bank debt. Finally, when g_t and B_t are both negative, the firm is increasing its cash buffer-stock.

To capture the debt-overhang effect, in the model there is a preexisting public debt obligation with a perpetual coupon $b > 0$.[3] The value of the public debt is denoted D. The public debt cannot be renegotiated. This assumption is made for two reasons. First, we want the model to capture the distortion posited by Myers (1977). Renegotiation would eliminate the distortion. Second, the assumption is adopted in the interest of realism. In the real world, the difficulty of renegotiating public debt stems from coordination costs, free-rider problems, and the unanimity requirement imposed by the Trust Indenture Act of 1939. Our model of public debt is also consistent with the empirical observation that private workouts are far less likely to succeed when the firm has public debt.

The second assumption imposed on the public debt is that it contains a covenant prohibiting the issuance of additional perpetual debt. Without such a prohibition, existing shareholders could make profits ex post by issuing additional perpetual debt that reduced the value of the claim held by current bondholders. Consistent with this assumption, Smith and Warner (1979) find that 90.8% of covenants in their sample contain some restriction on future debt issuance. However, the

[3] The investment optimality condition serving as the basis for our regression equation holds for *arbitrary* b. Moyen (2004) uses numerical optimization to choose the perpetual coupon. She obtains an analogous investment optimality condition.

stringency of covenants does vary. Covenants often limit asset sales as well. The model can be extended with another covenant mandating $I \geq 0$. Proposition 1 still holds if one imposes a non-negativity constraint on investment.

Assumption 3 summarizes the debt market.

Assumption 3 *In the event of default, absolute priority is obeyed and the firm is liquidated for $L(\varepsilon_T)K_T$. The bank is senior in priority. The interest rate on the bank credit line is r, with a collateral constraint ensuring the bank loan is risk-free. The firm earns rate $r_s < r$ on cash savings. The firm has a public debt obligation with perpetual coupon $b > 0$.*

2.1. Optimal financial and investment policies

The following budget constraint must be satisfied at each instant:

$$X_t = I_t + G(I_t, K_t) + b + \widetilde{r}B_t - g_t - F(K_t, \varepsilon_t). \tag{8}$$

The first four terms on the right side of the budget constraint Eq. (8) represent uses of funds: direct investment costs, indirect capital adjustment costs, coupon payments on public debt, and interest expense on the bank credit line. The last two terms in the budget constraint represent sources of funds other than external equity. The variable g measures new bank borrowing or reductions in the cash buffer stock, whereas F measures the cash inflow from current operations.

The vector $(K_t, B_t, \varepsilon_t)$ captures all relevant information at each point in time. At each instant, the manager chooses an optimal financial policy g_t and investment policy I_t. The manager must also choose an optimal time to default, denoted by the stopping time $T \in T$, where T is the set of all stopping times defined with respect to the natural filtration. In choosing the stopping rule T, the manager recognizes that the equity value function S is set equal to zero in the event of default. This is consistent with absolute priority being obeyed in bankruptcy. As is standard, the value function S pastes smoothly to zero along the optimal default region.

The manager solves

$$
\begin{aligned}
S(K_t, B_t, \varepsilon_t) &= \max_{g_{t+\tau}, I_{t+\tau}, T} \quad E_t\left[-\int_0^{T-t} e^{-r\tau} H(X_{t+\tau}, K_{t+\tau}) d\tau\right] \\
\text{subject to} &: \quad \text{Eqs.} \quad (2)\,, \quad (6)\,, \quad (7)\,, \quad \text{and} \quad (8)\,.
\end{aligned}
\tag{9}
$$

The solution to Eq. (9) is characterized using the Bellman equation,

$$
\begin{aligned}
rS &= \max_{g,I} \quad -H[I + G(I, K) + b + \tilde{r}B - g - F(K, \varepsilon), K] \\
&\quad +(I - \delta K)S_K + gS_B + \mu(\varepsilon)S_\varepsilon + \frac{1}{2}\sigma^2(\varepsilon)S_{\varepsilon\varepsilon} + \lambda[\gamma(L(\varepsilon)K - B) - g],
\end{aligned}
\tag{10}
$$

in which the relevant constraints are substituted directly into the objective function, with λ denoting the multiplier on the collateral constraint Eq. (7).

The first-order condition for the optimal financing policy is

$$-S_B = H_X - \lambda. \tag{11}$$

The complementary slackness condition yields a simple decision rule for optimal financing

$$\lambda = 0 \Longrightarrow -S_B = H_X \quad \textit{and} \tag{12}$$

$$\lambda > 0 \Longrightarrow -S_B < H_X \quad \textit{and} \quad g^* = \overline{g}. \tag{13}$$

The term $-S_B$ represents the shadow cost of bank debt, and the term H_X represents the marginal cost of equity finance. When the collateral constraint is slack, the optimal policy equates the two. When the collateral constraint is binding, the marginal cost of equity exceeds that on bank debt, because of rationing of credit.

The optimality condition determining investment is

$$S_K \equiv q = H_X * [1 + G_I(I^*, K)] = (-S_B + \lambda) * [1 + G_I(I^*, K)]. \tag{14}$$

The optimality condition Eq. (11) states that the firm invests up to the point that the shadow value of a unit of installed capital is just equal to the marginal cost of investment, taking into account the cost of funds. Rearranging terms, it is apparent that, conditional upon q, equity dependent firms invest less, with

$$G_I(I^*, K) = \frac{q}{H_X} - 1. \tag{15}$$

2.2. Implications for empirical testing

For the purpose of empirical testing, we assume the adjustment cost function G has the following functional form, consistent with Assumption 1:

$$G(I, K) = \frac{1}{2}\alpha K \left[\frac{I}{K} - \delta\right]^2, \tag{16}$$

in which $\alpha > 0$ is a parameter determining curvature. The functional form for G is common in the literature. For examples, see Summers (1981), Chirinko (1987), and Hennessy (2004).

Letting Φ be an indicator function for $X > 0$, we assume the external equity cost function has the following function form, consistent with Assumption 2:

$$H(X, K) = X + \Phi * \left[\frac{1}{2}\phi K \left(\frac{X}{K}\right)^2\right], \tag{17}$$

in which $\phi > 0$ is a parameter determining curvature.

Under the stated assumptions, the optimality condition for investment is

$$\begin{aligned} i^* &= \left(\delta - \frac{1}{\alpha}\right) + \left(\frac{1}{\alpha}\right)[1 + \Phi\phi x]^{-1} * q, && (18)\\ i &\equiv \frac{I}{K}, \quad and && (19)\\ x &\equiv \frac{X}{K}. && (20) \end{aligned}$$

Now define the function $f(\cdot)$ as

$$f(x) \equiv [1 + \phi\Phi x]^{-1}. \tag{21}$$

Conditional upon issuing equity, the median value of x (capital-normalized equity issuance) is 0.076 in our sample. A fair approximation of $f(x)$ can be obtained using a first-order Taylor expansion

$$f(x) \approx 1 - \phi\Phi x. \tag{22}$$

In the investment equations that follow, the $\approx$ sign stems from this linearization. Substituting this approximation into the optimality condition yields

$$i^* \approx \left(\delta - \frac{1}{\alpha}\right) + \left(\frac{1}{\alpha}\right) q - \left(\frac{\phi}{\alpha}\right)(q * \Phi x). \tag{23}$$

Eq. (23) states that, ceteris paribus, the rate of investment is higher for firms with high marginal q values. This is a standard feature of neoclassical models of investment under convex costs of adjusting the capital stock, e.g., Hayashi (1982). However, our model differs from the neoclassical model in that financing matters. In particular, the interaction term in Eq. (23) indicates that, conditional upon marginal q, an equity issuer invests less. Intuitively, costs of external equity discourage capital formation by raising the shadow cost of funds. Also, if marginal q were observable to the econometrician, the model would yield a simple prescription in terms of econometric testing. An interaction term between marginal q and normalized equity issuance should be included along with q as a regressor in the empirical investment equation.

Complicating the task of econometric estimation is the fact that marginal q is not directly observable. This is the subject of Proposition 1.

Proposition 1. *Under Assumptions 1-3, the shadow value of capital is*

$$q_t = Q_t - \frac{R_t}{K_t} + \frac{C_t}{K_t}, \tag{24}$$

$$Q_t \equiv \frac{S_t + B_t + D_t}{K_t}, \tag{25}$$

$$R_t \equiv E_t\left[e^{-r(T-t)} L(\varepsilon_T) K_T\right], \quad \textit{and} \tag{26}$$

$$C_t \equiv E_t\left[\int_0^{T-t} e^{-r\tau}\left[\lambda g_K K + (H_X - 1)(b + \widetilde{r}B - g)\right] d\tau\right]. \tag{27}$$

Proof. See Appendix A. ■

Under the perfect markets assumptions adopted by Hayashi (1982), there are no binding collateral constraints and the cost of a dollar of external equity is unity. In this case, $\lambda = 0$ and $H_X = 1$, implying that $C_t = 0$ in perfect markets. In Hayashi's model, the possibility of default never enters into the investment decision. In the notation of our model, a firm that never defaults treats T as infinite. For such a firm, the variable $R_t = 0$. Therefore, Proposition 1 is consistent with Hayashi's (1982) result that marginal q is equal to average Q in perfect financial markets. Otherwise, financial frictions drive a wedge between marginal and average Q.

The variable R is the correct empirical proxy for the Myers (1977) overhang effect. It measures the spillover benefit from capital accumulation accruing to lenders. R is large when T is low. Intuitively, for distressed firms with a small expected passage time to default, investment generates a large positive externality to lenders who recover the assets of the firm in bankruptcy. Also, holding T fixed, the variable R is large when the liquidation value of assets (L) is high. Intuitively, the discounted value of lender recoveries in default is capitalized into the market value of debt. However, lender recoveries in default have no effect on marginal q for shareholders. To determine true marginal q, these recovery values must be subtracted from average Q. Therefore, when L is high, a larger overhang adjustment is necessary.

Complicating econometric testing of the theory is the fact that the variable C cannot be observed directly. However, inspection of each of the terms suggests firm characteristics that are indicative of high values of C. The term λg_K captures the benefit of installed capital in terms of relaxing collateral constraints in future periods. Therefore, all else equal, C is high for firms facing binding collateral constraints in the future. Now consider the term

$$(H_X - 1)(b + \widetilde{r}B - g). \tag{28}$$

$H_X \geq 1$, with strict inequality when the firm issues equity. The term above is zero for healthy dividend-paying firms with $H_X = 1$. The term in Eq. (28) is highest for firms with high debt service obligations and equity dependent firms. Equity dependent firms are those relying heavily upon external equity for financing. To see this, note that an equity dependent firm has $H_X > 1$ and limited access to new debt (low g). The KZ and WW indices are reasonable proxies for firms with high C/K values.

Substituting the expression for q derived in Proposition 1 into the empirical investment Eq.

(23) yields

$$i^* \approx \left(\delta - \frac{1}{\alpha}\right) + \left(\frac{1}{\alpha}\right) Q - \left(\frac{\phi}{\alpha}\right) \underbrace{(Q * \Phi x)}_{InteractionTerm} - \left(\frac{1}{\alpha}\right) \underbrace{\left(\frac{R}{K}\right)}_{Overhang} \tag{29}$$
$$+ \left(\frac{1}{\alpha}\right) \underbrace{\left(\frac{C}{K}\right)}_{Index} + \left(\frac{\phi}{\alpha}\right) \left(\frac{R - C}{K} * \Phi x\right).$$

Thus the theory predicts that average Q and indices proxying for C/K have positive coefficients. The overhang correction (R/K) and the interaction term $(Q * \Phi x)$ are predicted to have negative coefficients.

These results are of particular interest because two of them are counterintuitive. First, casual intuition suggests that equity issuers invest more. However, the model predicts that conditionally an equity issuer invests less, because of costs of external equity. Fig. 1 provides some intuition to complement the formal proof provided above. Consider two firms, with the same value of $q > 1$, but assume that one of the firms has ample cash and that the second must fund all investment by floating new equity. For simplicity, we treat the depreciation rate (δ) as zero. Under the stated assumptions, the marginal cost investment for the unconstrained firm is

$$1 + G_I = 1 + \alpha i. \tag{30}$$

The unconstrained firm invests up to i_u^*, which is the point where the marginal benefit of investment (q) is just equal to the marginal cost. Under the stated assumptions, the constrained firm must finance with external equity with $i = x$. The marginal cost of investment for the constrained firm is

$$H_X(1 + G_I) = (1 + \phi i)(1 + \alpha i) = 1 + (\alpha + \phi)i + \alpha\phi i^2. \tag{31}$$

The constrained firm invests up to $i_c^* < i_u^*$. The difference in investment between the firms is the result of differing marginal costs of investment. In fact, both firms face convexity in the level of investment costs. However, the constrained firm also faces convexity in the marginal cost schedule. In contrast, the unconstrained firm faces a linear marginal cost schedule.

[Insert Fig.1 near here.]

The second prediction that is potentially counterintuitive is that a proxy for constrained status is predicted to have a positive coefficient. It is worth stressing that the model does not predict that constrained (equity dependent) firms invest more. Eq. (15) clearly shows that, conditional on q, a constrained firm is predicted to invest less. However, the model does predict that proxies for (C/K) have a positive coefficient when Eq. (29) is estimated. To complement the formal proof, we turn to Fig. 2 to provide the intuition. Let q_c^{naive} denote a naive estimate of marginal q for the constrained firm, ignoring the collateral effect (λg_K). The true marginal $q_c > q_c^{naive}$. We can view the manager as using a two-step procedure in deciding how much to invest. In step one, he ignores the collateral value of investment and chooses i_c^{naive}. In step two, he increases his investment to i_c^* taking into account the collateral effect. The variable C/K is effectively capturing this second subtle effect.

[Insert Fig.2 near here.]

Although the focus of the paper is investment, it is assuring to note that the model generates reasonable predictions regarding the financial behavior of the firm. The optimal financial policy hinges upon S_B, which is the shadow cost of increasing the bank credit line or reducing the cash buffer-stock. Proposition 2 is informative about the factors determining the magnitude of this key variable.

Proposition 2. *The shadow cost of debt is*

$$S_B = E\left[\int_0^T e^{-r\tau}[\lambda g_B - \widetilde{r} H_X] d\tau\right] < 0. \tag{32}$$

Proof. See Appendix A. ■

In this model two types of firms are predicted to pay dividends. First, consider a firm that is likely to default in the near future. As T approaches zero, so too does $|S_B|$. From the optimality condition Eq. (11) it follows that the firm faces a binding collateral constraint. Intuitively, a firm nearing default pays out any cash from its buffer-stock and then exhausts its credit line. This sort of behavior is the financial analog of the Myers overhang effect for real investments. Real-world contract provisions can be viewed as placing other direct constraints on the firm in light of such incentives. For example, bank credit lines often contain clauses making it difficult or impossible for a firm to draw on its credit line if it has suffered a material adverse change.[4]

The second type of firm predicted to pay dividends is a corporation such as Microsoft, with an ample cash buffer-stock. Such a firm pays out just enough cash to ensure that the shadow value of a unit of cash on hand, as measured by $|S_B|$, is just equal to unity. The precautionary motive for cash retentions, as captured by the term λg_B, is balanced against low rate of return earned on retained cash $\widetilde{r} = r_s < r$.

When $|S_B| > 1$, the optimality condition in Eq. (11) implies that the firm floats some amount of new equity. Inspecting Proposition 2, such a firm must have a fairly long horizon (T), must anticipate being a borrower the majority of the time ($\widetilde{r} = r$), and must anticipate binding collateral constraints in the future. Effectively, the firm is smoothing the costs of external equity.

3. Data and summary statistics

The data come from two sources. The first is the combined annual, research, and full coverage 2005 Standard and Poor's Compustat industrial files. The sample is selected by first deleting any firm-year observations with missing data or for which total assets, the gross capital stock, or sales are either zero or negative. All observations that fail to obey standard accounting identities are deleted. A firm is included in the sample only if it has at least three consecutive years of complete Compustat data. Finally, a firm is omitted if its primary standard industrial classification (SIC) is between 4900 and 4999, between 6000 and 6999, or greater than 9000, because the model is inappropriate for regulated, financial, and public service firms.

Some of the data variables require calculation of market betas and the standard deviation of

[4]Such a constraint would not affect the investment optimality condition that we estimate.

the residual from the market model. To do so we use monthly stock returns from the 2005 Center for Research in Security Prices (CRSP) tapes. For a firm-year observation to be included in the sample, the firm must have at least three years of complete return data preceding the year that the firm is in the Compustat sample. Our market betas and residual standard deviations are calculated using from three to five years of data, depending on availability. After merging the CRSP and Compustat data, we end up with a total of 46,118 firm-year observations from 1968 to 2003, with a minimum of 657 observations per year and a maximum of 1723 observations per year. A detailed description of the variables is in Appendix B.

The debt-overhang correction represents the current value of lenders' rights to recoveries in default. Recovery ratios by SIC are based on Altman and Kishore (1996). We ignore short-term debt and assume that long-term debt matures in a straight-line fashion, with 5% of the original debt maturing each year.[5] This allows us to estimate R_t/K_t as

$$R_t/K_t = \mathit{Leverage}_t * \mathit{Recovery\ Ratio} * [\sum_{s=1}^{20} \rho_{t+s}[1 - .05(s-1)](1+r)^{-s}], \tag{33}$$

in which ρ_{t+s} denotes the probability of default in period t+s based on the date t debt rating. Moody's reports of hazard rates by bond rating are used as the basis for default probabilities. When bond ratings were not available in Compustat, they were imputed based on the estimates from Blume, Lim, and MacKinlay (hereafter, BLM, 1998).

Because the variable C/K includes the benefit of installed capital in terms of relaxing collateral constraints in future periods, a good proxy identifies firms that are likely to face a binding constraint. The first proxy we employ is the widely used KZ index. This index comes from Kaplan and Zingales (1997), who examine the annual reports of the 49 firms in the Fazzari, Hubbard, and Petersen (1988) constrained sample, using this information to rate the firms on a financial constraints scale from one to four. They then run an ordered logit of this scale on observable firm characteristics. Several authors use these logit coefficients on data from a broad sample of firms to construct a synthetic KZ index to measure finance constraints. Specifically, the

[5]This approximation implies a weighted-average maturity of 10.5 years and is close to that found in Klock, Thies, and Baum (1991), who estimate the average term to maturity was 15.3 years in 1977 and 12.7 years in 1983 for a sample of one hundred manufacturing firms.

KZ index is constructed as

$$-1.001909CF + 3.139193TLTD - 39.36780TDIV - 1.314759CASH + 0.2826389Q, \tag{34}$$

in which CF is the ratio of cash flow to book assets, $TLTD$ is the ratio of total long-term debt to book assets, $TDIV$ is the ratio of total dividends to book assets, $CASH$ is the ratio of the stock of cash to book assets, and Q is the market-to-book ratio. Examination of the index reveals that it is likely to isolate firms with a high C/K ratio, those with low cash, low cash flow, and high debt burdens.

Following Baker, Stein, and Wurgler (2003), we exclude the Q term when computing the pseudo-KZ index for each firm. There are two reasons for doing so. First, Q enters the empirical regression equation separately, and we want to isolate its effect from the effect of collateral constraints. Second, Whited (2001) has demonstrated that, in its role as a proxy for marginal q, the market-to-book ratio contains a great deal of measurement error.

The second measure of a binding collateral constraint is from Whited and Wu (2006). Their index is constructed from the Euler equation from a standard intertemporal investment model with convex adjustment costs. In this model, and in the absence of constraints, the marginal cost of investing today equals the opportunity cost of postponing investment until tomorrow. In the presence of constraints, a wedge appears between these two costs. The WW index is an estimated parameterization of this wedge. Specifically, the WW index is

$$-0.091CF - 0.062DIVPOS + 0.021TLTD - 0.044LNTA + 0.102ISG - 0.035SG. \tag{35}$$

Here, $DIVPOS$ is an indicator that is one if the firm pays dividends, and zero otherwise; SG is own-firm real sales growth; ISG is three-digit industry sales growth; and $LNTA$ is the natural log of book assets. Examination of this index indicates it is likely to identify firms with a high C/K ratio. Firms with a high WW index are small, have high debt burdens, and low cash flow. Also, they are the slow-growing firms in fast-growing industries.

Table 1 presents summary statistics from the sample. The first two rows describe the full sample. Of interest here are the figures for equity issuance, which occurs in 17.9% of the firm-years. The next two lines describe separately the firms that issue equity and those that do not. Not surprisingly, the equity issuers are small, invest at a high rate, and have high Q values. They also have low cash flow relative to investment, high total debt, but low debt net of cash. The rest of the table examines the characteristics of firms categorized as constrained and unconstrained according to the KZ and WW indices. In reading the table, recall that our empirical testing is predicated upon the assumption that a high value for either index is indicative of the firm having a high C/K ratio.

[Insert Table 1 near here.]

The firms categorized as constrained according to the KZ index do not appear to be more equity dependent than average, and they do not appear to be much smaller than average. They also seem to be investing at an average rate, despite a lower than average Tobin's q. Although these characteristics do not point to the types of firms that face collateral constraints, other characteristics do. The high KZ firms do not have enough cash flow to cover investment and they are highly indebted. These last two characteristics are consistent with a binding collateral constraint. As expected, the firms classified as unconstrained have far more cash flow than investment, as well as low leverage.

The firms categorized as constrained according to the WW index are clearly equity dependent: They issue equity more often than average and, when they issue, the size of the issue (relative to the size of the firm) is larger than average. They also have somewhat higher leverage than average. This dependence on outside finance makes sense because these firms have low cash flow relative to investment. In contrast, firms with a low WW index have much higher cash flow than investment and depend on outside finance less than average. In sum, these two indices do appear to identify firms that could benefit from a relaxation of a collateral constraint. The main difference between the two types of constrained firms is that the KZ-constrained firms rely more on debt and the WW-constrained firms rely more on equity. This result primarily stems from the heavy loading on industry sales growth in the WW index, which is not present in the KZ index. Firms in fast-growing industries are much more likely to finance with equity.

4. Empirical tests

Table 2 presents results from estimating the investment regression in Eq. (29) via ordinary least squares (OLS).[6] As is standard in the literature, the model is estimated using firm and year-specific intercepts. A number of factors can generate fixed effects. For example, in Eq. (29), depreciation rates (δ) differs across firms and over time, affecting the optimal rate of investment. Consistent with this view, a standard Hausman test rejects a specification without firm- and year-specific intercepts.

[Insert Table 2 near here.]

The first two lines contain estimated coefficients from baseline specifications in which the KZ and WW indices are used as proxies for (C/K). All coefficients are consistent with the model's predictions, and all coefficients are significant at the 5% level. First, we note that the simple Q theory is easily rejected, because regressors other than Q enter significantly.

The coefficient on the interaction term is negative, indicating that, conditional on average Q, equity issuers invest less. This is consistent with corporations perceiving convex costs of external equity. The theoretical models of Myers and Majluf (1984) and Krasker (1986) demonstrate that such costs can be traced to asymmetric information problems in the equities market.

The coefficient on the overhang correction, (R/K), is negative, indicating that heavily levered firms with high probabilities of default invest less. This lends support to the Myers (1977) contention that firms tend to underinvest because of the failure of managers to account for investment returns accruing to lenders. Also on average estimates of the magnitude of the coefficients on the overhang corrections imply that the elasticity of investment with respect to leverage for the median firm is approximately -1. This substantial effect suggests that the Myers

[6] The final term interacting $(R-C)/K$ and Φx is omitted. It is never significant and has no material effect.

underinvestment problem is more than a theoretical curiosity. Instead, debt overhang creates a large drag on the investment of distressed firms.

Finally, the coefficient on either proxy for (C/K) is positive and significant. This indicates that a collateral channel is operative. In particular, firms that are more likely to face binding collateral constraints and firms that are more equity dependent appear to account for the positive spillover that investing today provides in terms of facilitating access to collateralized loans. This effect is consistent with the models of Hart and Moore (1994) and Kiyotaki and Moore (1997), which stress that loans could be rationed based on collateral value. As discussed in Section 1, these findings complement those in Almeida and Campello (2004), who present evidence favoring the existence of a collateral effect.

4.1. Robustness

In the remainder of the analysis, we address the three sources of concern that have been most commonly cited in the investment literature. The first concern is the possibility of endogeneity bias in the estimates in Table 2. However, this concern is likely to be of little importance for two reasons. First, Q and the overhang correction are measured at the beginning of the year, whereas investment is measured during the course of the year. Therefore, although not strictly exogenous, these two right-hand-side variables can be considered predetermined. A similar argument applies to several of the components of the KZ and WW indices; and one of the components of the WW index, industry sales growth, is easily argued to be an exogenous variable. The predetermined or exogenous components of the KZ and WW indices account for 61% and 70% of their total variation, respectively. Second, any endogeneity bias in the coefficients on the interaction term or the overhang correction is likely to work against finding a negative coefficient. To see this, note that the regression disturbance can be interpreted as an unobservable shock to investment that is orthogonal to Q. A positive realization of such a shock increases both investment and either debt or equity issuance, thus resulting in a positive correlation between the regression disturbance and either the interaction term or the debt-overhang term. These positive correlations result in coefficients that are biased upward. Therefore, if endogeneity bias is important, the true coefficients on the interaction term and on the overhang correction are even more negative than the coefficients estimated via OLS.

The second concern commonly cited in the literature is bias resulting from curvature of the profit function. Hayashi (1982) notes that, even in perfect financial markets, average Q is not a sufficient statistic for investment when linear homogeneity (Assumption 1) is violated. For example, the profit function (F) is not linear homogeneous when the firm has market power or decreasing returns. Cooper and Ejarque (2003) and Abel and Eberly (2004) show that cash flow proxies for the wedge between marginal q and average Q when the firm has market power. Thus market power provides a plausible explanation for the significance of cash flow documented in numerous papers.

To address concern over market power or decreasing returns, the third and fourth regressions in Table 2 add cash flow to the regression. The addition of cash flow has a small effect. All coefficients have the predicted signs. The coefficient on the overhang correction increases in absolute value and remains significant at the 5% level. The coefficient on the KZ index increases and remains significant at the 5% level. The coefficient on the WW index falls but remains significant at the 5% level. The interaction term continues to have the predicted negative sign, but is only significant at the 10% level. Summing up, it appears that market power and decreasing returns are not driving the results.

Cash flow enters significantly, which is inconsistent with the model as stated. There are three plausible explanations for the significance of cash flow. First, cash flow could be picking up curvature in the profit function resulting from market power or decreasing returns. Second, the significance of cash flow could be the result of measurement error in Q. Finally, cash flow effects could be the result of managerial empire building à la Jensen (1986).

The third commonly cited source of concern in the investment literature is measurement error in average Q. Poterba (1988) argues that the inevitable measurement errors in average Q biases coefficients, and Erickson and Whited (2000) delineate possible sources of measurement error. Consistent with this view Erickson and Whited (2000) use measurement-error consistent GMM in a regression of investment on Q and cash flow, finding that only 40% of the variation in observable values of Q stems from variation in true Q. They also find that Q increases in importance and cash flow decreases in importance after controlling for measurement error.[7]

Based on this evidence, it is reasonable to question whether measurement error in Q is driving the reported results. To address this concern, we derive a GMM estimator that is consistent even when the first regressor, average Q, is measured with noise. (See Appendix B.) Essentially, we tailor the estimators for the linear model in Erickson and Whited (2000, 2002) to accommodate the particular nonlinear specification in Eq. (29). There exists no off-the-shelf remedy for measurement error in nonlinear models, because the appropriate correction is always model specific. For example, the remedy in Hausman, Ichimura, Newey, and Powell (1991) assumes a specific polynomial functional form, and the method in Schennach (2004) requires extra information not available in Compustat. Further, as discussed in Erickson and Whited (2000), measurement error in Q is likely to be serially correlated. Therefore, lagged instrumental variables are inappropriate.[8]

Table 3 reports coefficients using measurement-error consistent GMM to estimate the model. The coefficients on average Q remain significant and are over twice as large as their OLS counterparts. For the median firm in our sample, the implied elasticities of investment with respect to Q rise from approximately 0.13 to 0.42. Similarly, all but one of the coefficients on the interaction term and all of the coefficients on the overhang correction remain significantly negative. The correction for measurement error produces a rise in the absolute magnitudes of these coefficients. For example, whereas the OLS estimates on the interaction term are small in magnitude, according to the estimates in Table 3 the median equity-issuing firm in our sample would increase investment by approximately 5% if it were to finance instead with internal funds.

[7] The Erickson and Whited (2000, 2002) estimators are also used in Hennessy (2004) and Almeida and Campello (2004).

[8] Measurement error clearly exists in the two proxies for (C/K) as well, because this term is inherently unobservable. Dealing with two mismeasured variables in a nonlinear model is an intractable problem. Therefore, it is important to interpret the estimated coefficients on the proxies for (C/K) literally, as if taking the KZ and WW indices at face value. On the one hand, this strategy alleviates the measurement error problem, because variables that are taken at face value cannot, by definition, contain measurement error. On the other hand, the estimated coefficients cannot be given a strict structural interpretation.

Also, the elasticity of investment with respect to the overhang correction more than doubles in comparison with the OLS estimates. The coefficients on the two indices remain positive and significant, with those on the WW index decreasing and those on the KZ index increasing. Although the coefficients on cash flow remain significantly greater than zero, they are smaller than the OLS estimates and much smaller than figures reported in most of the literature. For example, the cash-flow coefficient for the unconstrained group of firms in the original Fazzari, Hubbard, and Petersen study is 0.254, and the coefficient for the most constrained group is 0.670. Finally, the measurement-error consistent estimates of the regression R^2s are substantially higher than in the OLS case.[9]

Table 4 contains a final robustness check. Because both the WW and KZ indices contain cash flow as a component, while cash flow enters the regression separately, the cash flow coefficients reported in Tables 2 and 3 could be biased. Further, the presence of measurement error in Q can cause this bias to spill over to other regressors, because Q is correlated with all of the variables in the regression. To examine this issue, we strip cash flow out of both indices and re-estimate the models using both OLS and GMM. The results are similar, suggesting that this concern does not drive the findings.

In summary, the evidence is consistent with the predictions of the model. That is, corporate investment is affected by convex costs of external equity (negatively), debt overhang (negatively), and collateral effects (positively). The statistical significance of the latter two channels is highest.

5. Conclusion

This paper provides an internally consistent framework for empirically examining the effects of financial frictions on investment. This endeavor is a contribution to a literature that has based most of its tests on embedding predictions from a static investment model into a regression of investment on Q, which is itself based on a dynamic model. For example, the most commonly used test consists of inserting cash flow into such a regression and examining whether a priori

[9] Although not reported, the overidentifying restrictions of the model are rejected at the 10% level in only two out of the 36 cross sections. This evidence bolsters the hypothesis that endogeneity is not important, because the moment equations would be violated in the presence of a correlated error and regressor.

constrained firms have large significant coefficients on cash flow. This strategy follows from static models in which financially constrained firms invest more after experiencing cash windfalls. However, these sorts of tests can be misleading, because, as in Gomes (2001), features that generate a certain prediction in a static model can generate a different prediction when completely integrated into a dynamic optimizing framework.

In contrast, the model developed here is one in which a firm optimizes over time and under uncertainty, while facing three important frictions: convex costs of external equity, collateral constraints, and debt overhang. The model produces several interesting predictions, two of which are counterintuitive. First, although one would expect firms issuing equity to have high investment, firms issuing equity invest less, conditional on Q, because of the extra costs associated with floating equity. Second, conditional on Q, firms that expect to face collateral constraints in the future invest more today, not less. This sort of preemptive investment occurs because installed capital relaxes collateral constraints. The final prediction of the model, which is intuitive, is that firms experiencing overhang invest less. The model shows that all of these predictions can be nested into a single regression of investment on Q and three extra terms representing each of the above frictions. Evidence from data on US firms supports the model, and this evidence appears to be robust to measurement error in Q.

Although the model and empirical tests embody a rich menu of financing frictions, in the interest of tractability our analysis abstracts from agency conflicts between managers and shareholders. A challenge for future research is using dynamic models to understand the effects on investment of agency conflicts between managers and shareholders.

Appendix A

Proof of roposition 1.

The Bellman equation holds point-wise, implying that partial derivatives of both sides of Eq. (10) must match. Differentiating with respect to K yields

$$\begin{aligned} rS_K &= -H_X X_K - H_K + (I - \delta K)S_{KK} + S_K(I_K - \delta) + gS_{KB} + S_B g_K + \\ &\quad \mu(\varepsilon)S_{K\varepsilon} + \frac{1}{2}\sigma^2(\varepsilon)S_{K\varepsilon\varepsilon} + \lambda(\gamma L - g_K) + [\gamma(LK - B) - g]\lambda_K. \end{aligned} \tag{36}$$

In the interest of brevity, define the Dynkin's operator on a twice differentiable function, say f, as

$$A(f) \equiv (I - \delta K)f_K + gf_B + \mu(\varepsilon)f_\varepsilon + \frac{1}{2}\sigma^2(\varepsilon)f_{\varepsilon\varepsilon}. \tag{37}$$

Using the operator A and the definition of q, Eq. (36) can be simplified as

$$rq = -H_X X_K - H_K + A(q) - \delta q + qI_K + S_B g_K + \lambda(\gamma L - g_K) + [\gamma(LK - B) - g]\lambda_K. \tag{38}$$

Substituting X_K and the optimality conditions, the equation above can be rewritten as

$$rq = H_X(F_K - G_K) - H_K + A(q) - \delta q + \lambda\gamma L + [\gamma(LK - B) - g]\lambda_K. \tag{39}$$

The following complementary slackness condition is satisfied under the optimal program

$$\lambda[\gamma(LK - B) - g] = 0. \tag{40}$$

Because the complementary slackness condition must also be satisfied point-wise, the partial derivative of the left side of Eq. (40) must be zero. This implies

$$\lambda\gamma L + [\gamma(LK - B) - g]\lambda_K = \lambda g_K. \tag{41}$$

Substituting Eq. (41) into Eq. (40) and multiplying through by K yields

$$rqK = H_X(F_K K - G_K K) - H_K K + KA(q) - \delta qK + \lambda g_K K. \tag{42}$$

From Ito's lemma and the optimality condition on I it follows that

$$KA(q) - \delta qK = A(qK) - qI = A(qK) - H_X(I + G_I I). \tag{43}$$

Substituting Eq. (43) into Eq. (42) and invoking homogeneity of F and G (Assumption 1) yields

$$rqK = -H_X(I + G - F) - H_K K + \lambda g_K K + A(qK). \tag{44}$$

From the definition of X, the equation above can be rewritten as

$$rqK = -H_X X - H_K K + \lambda g_K K + A(qK) + H_X(b + \widetilde{r}B - g). \tag{45}$$

Invoking homogeneity of H (Assumption 2), the equation above can be rewritten as

$$rqK = -H + \lambda g_K K + A(qK) + H_X(b + \widetilde{r}B - g). \tag{46}$$

The Bellman equation can be written in a symmetric form as

$$rS = -H + A(S). \tag{47}$$

Subtracting Eq. (46) from Eq. (47) and rearranging terms yields

$$A(S - qK) - r(S - qK) = \lambda g_K K + H_X(b + \widetilde{r}B - g). \tag{48}$$

In the remainder of the derivation, subscripts denote time, not partial derivatives. Also, time is normalized such that date zero is the present. Using Dynkin's formula (Oksendal, Theorem 7.4.1), we have

$$E[e^{-rT}(S_T - q_T K_T)] = S_0 - q_0 K_0 + E\left[\int_0^T e^{-rt}[A(S - qK) - r(S - qK)]dt\right]. \tag{49}$$

The smooth-pasting conditions for optimal default are $S_T = q_T = 0$. Substituting the smooth-pasting conditions and Eq. (48) in Eq. (49) yields

$$q_0 K_0 = S_0 + E\left[\int_0^T e^{-rt}[\lambda g_K K + H_X(b + \widetilde{r}B - g)]dt\right]. \tag{50}$$

Now make the following substitution

$$H_X(b + \widetilde{r}B - g) = (H_X - 1)(b + \widetilde{r}B - g)] + (b + \widetilde{r}B - g) \tag{51}$$

and rearrange terms to obtain

$$\begin{aligned} q_0 K_0 \quad = \quad & S_0 + E\left[\int_0^T e^{-rt} b dt\right] + E\left[\int_0^T e^{-rt}[\widetilde{r}B - g]dt\right] + \\ & E\left[\int_0^T e^{-rt}[\lambda g_K K + (H_X - 1)(b + \widetilde{r}B - g)]dt\right]. \end{aligned} \tag{52}$$

The rest of the proof follows from substituting the following bond pricing identities into Eq. (52)

$$
\begin{aligned}
B_0 &\equiv E\left[\int_0^T e^{-rt}[\widetilde{r}B - g]dt + e^{-rT}B_T\right], && (53) \\
D_0 &\equiv E\left[\int_0^T e^{-rt}bdt + e^{-rT}D_T\right], \quad \textit{and} && (54) \\
L_T K_T &= B_T + D_T. \blacksquare && (55)
\end{aligned}
$$

Proof of *Proposition 2.*

The proof is similar to that of Proposition 1. For the purpose of the proof, let $m \equiv S_B$. Differentiating both sides of the Bellman equation with respect to B yields

$$
rm = -H_X X_B + A(m) + qI_B + mg_B - \lambda(\gamma + g_B) + [\gamma(LK - B) - g]\lambda_B. \tag{56}
$$

Using the optimality conditions, the equation above simplifies as

$$
rm = -\widetilde{r}H_X - \gamma\lambda + [\gamma(LK - B) - g]\lambda_B + A(m). \tag{57}
$$

Differentiating the complementary slackness condition with respect to B yields

$$
-\gamma\lambda + [\gamma(B - LK) - g]\lambda_B = \lambda g_B. \tag{58}
$$

Substituting the equation above into Eq. (57) yields

$$
A(m) - rm = \widetilde{r}H_X - \lambda g_B. \tag{59}
$$

The rest of the proof follows from substituting the equation above and the smooth pasting condition $m_T = 0$ into Dynkin's formula:

$$
E[e^{-rT}m_T] = m_0 + E\left[\int_0^T e^{-rt}[A(m) - rm]dt\right]. \blacksquare \tag{60}
$$

Appendix B

This appendix contains details of the data construction. Basic Compustat data variables are defined as follows: Book assets are Compustat Item 6; gross capital stock is Item 7; investment is the difference between Items 30 and 107; cash flow is the sum of Items 18 and 14; equity issuance is Item 108 minus Item 115; total long-term debt is Item 9 plus Item 34; total dividends is Item 19 plus Item 21; stock of cash is Item 1; and sales is Item 12. Net debt is total long-term debt less cash. Average Q is calculated as in Erickson and Whited (2000). The market-to-book ratio is calculated as [(Item 6 + (Item 24) $\times$ (Item 25) $-$ Item 60 $-$ Item74)/Item 6].

The imputation of bond ratings using the method in Blume, Lim, and Mackinlay (1998) starts with the computation of average interest coverage [(Item 178 + Item 15)/Item 15], the operating margin (Item 13/Item 12), long-term debt leverage [(Item 34 + Item 9 + Item 104)/Item 6], the natural log of the market value of equity deflated to 1978 by the Consumer Price Index [(Item 24) $\times$ (Item 25)], beta, and standard error over the previous three to five years. Z scores were then obtained by using the coefficients on these variables and year dummies. The final imputed bond ratings were calculated by applying the cutoff values provided in Blume, Lim, and Mackinlay (1998) to these Z scores.

Appendix C

This appendix derives the estimators used to construct Tables 3 and 4. To describe the basic econometric framework, we rewrite Eq. (29) as

$$\tilde{y} = \tilde{\chi}\beta_1 + \tilde{\omega}\beta_2 + v\beta_3 + u, \tag{61}$$

where $\tilde{y}$ is the rate of investment, $\tilde{\chi}$ is unobserved true Q, $\tilde{\omega}$ is the interaction between $\tilde{\chi}$ and equity issuance, v is a vector including the other regressors, and u is a disturbance term.

The relation between $\tilde{\chi}$ and its observed proxy $\tilde{\xi}$ can be written as

$$\tilde{\xi} = \tilde{\chi} + e, \tag{62}$$

in which e is the measurement error in $\tilde{\xi}$. Let $\tilde{\mathrm{w}}$ be the observed counterpart of $\tilde{\omega}$, and let z be equity issuance. Then Eq. (62) immediately implies

$$\tilde{\mathrm{w}} = \tilde{\omega} + ze. \tag{63}$$

The regression disturbance, u, and the measurement error, e, are assumed independent of one another and of $(\tilde{\chi}, \tilde{\omega}, z)$. The vector (χ, ω, z, u, e) is assumed to be independently and identically distributed (i.i.d.).

To simplify computations, first partial out the perfectly measured variables in Eqs. (61)–(63) and rewrite the resulting expressions in terms of population residuals. This yields

$$y = \chi\beta_1 + \omega\beta_2 + u, \tag{64}$$

$$\xi = \chi + e, \quad and \tag{65}$$

$$\mathrm{w} = \omega + ze. \tag{66}$$

Here $y \equiv \tilde{y} - v\mu_y$, $\chi \equiv \tilde{\chi} - v\mu_x$, $y \equiv \tilde{y} - v\mu_y$, $\omega \equiv \tilde{\omega} - v\mu_w$, $\xi \equiv \tilde{\xi} - v\mu_x$, and $\mathrm{w} \equiv \tilde{\mathrm{w}} - v\mu_w$, with $\left(\mu_y, \mu_x, \mu_w\right) \equiv \left[E\left(v'v\right)\right]^{-1} E\left[v'\left(\tilde{y}, \tilde{\xi}, \tilde{\mathrm{w}}\right)\right]$. By substituting the OLS estimates of $\left(\mu_y, \mu_x, \mu_w\right)$ into Eqs. (64), (65), and (66), estimates of β, $E\left(u^2\right)$, $E\left(e^2\right)$, $E\left(\chi^2\right)$, $E\left(\omega^2\right)$, and $E\left(\chi\omega\right)$ can be obtained with the GMM procedure described in the next paragraph. Estimates of the j-th element of β_3 are obtained by substituting the GMM estimate of (β_1, β_2) and the j-th elements of the estimates of $\left(\mu_y, \mu_x, \mu_w\right)$ into

$$\beta_{3j} = \mu_{yj} - \mu_{xj}\beta_{1j} - \mu_{wj}\beta_{2j}. \tag{67}$$

An estimate of $\rho^2 \equiv 1 - \mathrm{var}(u)/\,\mathrm{var}(y)$, the population R^2 for Eq. (61), is obtained by evaluating

$$\rho^2 = \frac{\mu_y' \,\mathrm{var}(v)\mu_y + E\left(\chi^2\right)\beta_1^2 + E\left(\omega^2\right)\beta_2^2 + 2\beta_1\beta_2\left(\chi\omega\right)}{\mu_y' \,\mathrm{var}(v)\mu_y + E\left(\chi^2\right)\beta_1^2 + E\left(\omega^2\right)\beta_2^2 + 2\beta_1\beta_2\left(\chi\omega\right) + E\left(u^2\right)} \tag{68}$$

at the OLS estimate of μ_y, the sample covariance matrix for v, and the GMM estimates of β_1, β_2, $E\left(\chi^2\right)$, $E\left(\omega^2\right)$, $E\left(\chi\omega\right)$, and $E\left(u^2\right)$.

This model is nonlinear in the mismeasured regressor and cannot, therefore, be estimated via methods designed for linear models, such as that in Erickson and Whited (2002). Nonetheless, in a manner similar to that in Erickson and Whited (2002), this model can be estimated using the second and third moments of the observable variables. The first moment equation can be derived by squaring both sides of Eq. (64), setting them equal to one another, and then taking expectations of both sides. The result is:

$$E\left(y^2\right) = 2\beta_2\beta_1 E\left(\chi\omega\right) + E\left(u^2\right) + \beta_1^2 E\left(\chi^2\right) + \beta_2^2 E\left(\omega^2\right). \tag{69}$$

The second moment equation can be derived analogously by multiplying Eq. (64) by Eq. (65) and taking expectations of both sides:

$$E\left(y\xi\right) = \beta_2 E\left(\chi\omega\right) + \beta_1 E\left(\chi^2\right). \tag{70}$$

Continuing in this fashion results in an overidentified system of 19 equations in 18 unknowns.

$$\begin{aligned}
E\left(y^2\right) &= 2\beta_2\beta_1 E\left(\chi\omega\right) + E\left(u^2\right) + \beta_1^2 E\left(\chi^2\right) + \beta_2^2 E\left(\omega^2\right), && (71)\\
E\left(y\xi\right) &= \beta_2 E\left(\chi\omega\right) + \beta_1 E\left(\chi^2\right), && (72)\\
E\left(y\mathrm{w}\right) &= \beta_1 E\left(\chi\omega\right) + \beta_2 E\left(\omega^2\right), && (73)\\
E\left(\xi^2\right) &= E\left(\chi^2\right) + E\left(e^2\right), && (74)\\
E\left(\xi\mathrm{w}\right) &= E\left(\chi\omega\right) + E\left(z\right) E\left(e^2\right), && (75)\\
E\left(\mathrm{w}^2\right) &= E\left(\omega^2\right) + E\left(z^2\right) E\left(e^2\right), && (76)\\
E\left(y^2\xi\right) &= 2\beta_2\beta_1\left(\chi^2\omega\right) + \beta_1^2\left(\chi^3\right) + \beta_2^2\left(\chi\omega^2\right), && (77)\\
E\left(y\xi^2\right) &= \beta_1\left(\chi^3\right) + \beta_2\left(\chi^2\omega\right), && (78)\\
E\left(\xi^2\mathrm{w}\right) &= E\left(z\right) E\left(e^3\right) + E\left(\chi^2\omega\right) + 2E\left(z\chi\right) E\left(e^2\right), && (79)\\
E\left(\xi\mathrm{w}^2\right) &= E\left(\chi\omega^2\right) + 2E\left(e^2\right) E\left(z\omega\right) + E\left(z^2\right) E\left(e^3\right) + E\left(z^2\chi\right) E\left(e^2\right), && (80)\\
E\left(y^2\mathrm{w}\right) &= 2\beta_2\beta_1 E\left(\chi\omega^2\right) + \beta_2^2 E\left(\omega^3\right) + \beta_1^2 E\left(\chi^2\omega\right), && (81)\\
E\left(y\mathrm{w}^2\right) &= \beta_2 E\left(\omega^3\right) + \beta_1 E\left(\chi\omega^2\right) + \beta_1 E\left(z^2\chi\right) E\left(e^2\right) + \beta_2 E\left(z^2\omega\right) E\left(e^2\right), && (82)
\end{aligned}$$

$$
\begin{aligned}
E(z) &= E(z), & (83)\\
E\left(z^2\right) &= E\left(z^2\right), & (84)\\
E(z\xi) &= E(z\chi), & (85)\\
E(z\mathrm{w}) &= E(z\omega), & (86)\\
E(zy) &= E(z\chi)\beta_1 + E(z\omega)\beta_2, & (87)\\
E\left(z^2\xi\right) &= E\left(z^2\chi\right), \quad \textit{and} & (88)\\
E\left(z^2\mathrm{w}\right) &= E\left(z^2\omega\right). & (89)
\end{aligned}
$$

The parameters to be estimated are β_1, β_2, $E\left(\chi^2\right)$, $E\left(u^2\right)$, $E\left(\omega^2\right)$, $E(\chi\omega)$, $E\left(e^2\right)$, $E\left(\chi^2\omega\right)$, $E\left(\chi^3\right)$, $E\left(\chi\omega^2\right)$, $E\left(e^3\right)$, $E\left(\omega^3\right)$, $E(z\chi)$, $E(z\omega)$, $E\left(z^2\chi\right)$, $E\left(z^2\omega\right)$, $E(z)$, and $E\left(z^2\right)$. As in Erickson and Whited (2002), this model can be estimated by using GMM to pool the information contained in the second- and third-order moments of the observable variables. Because of the i.i.d. assumption, the system is estimated for each cross section. Pooled estimates are then obtained by averaging the cross-sectional estimates over the sample period. Standard errors are calculated using the method of Fama and MacBeth (1973).

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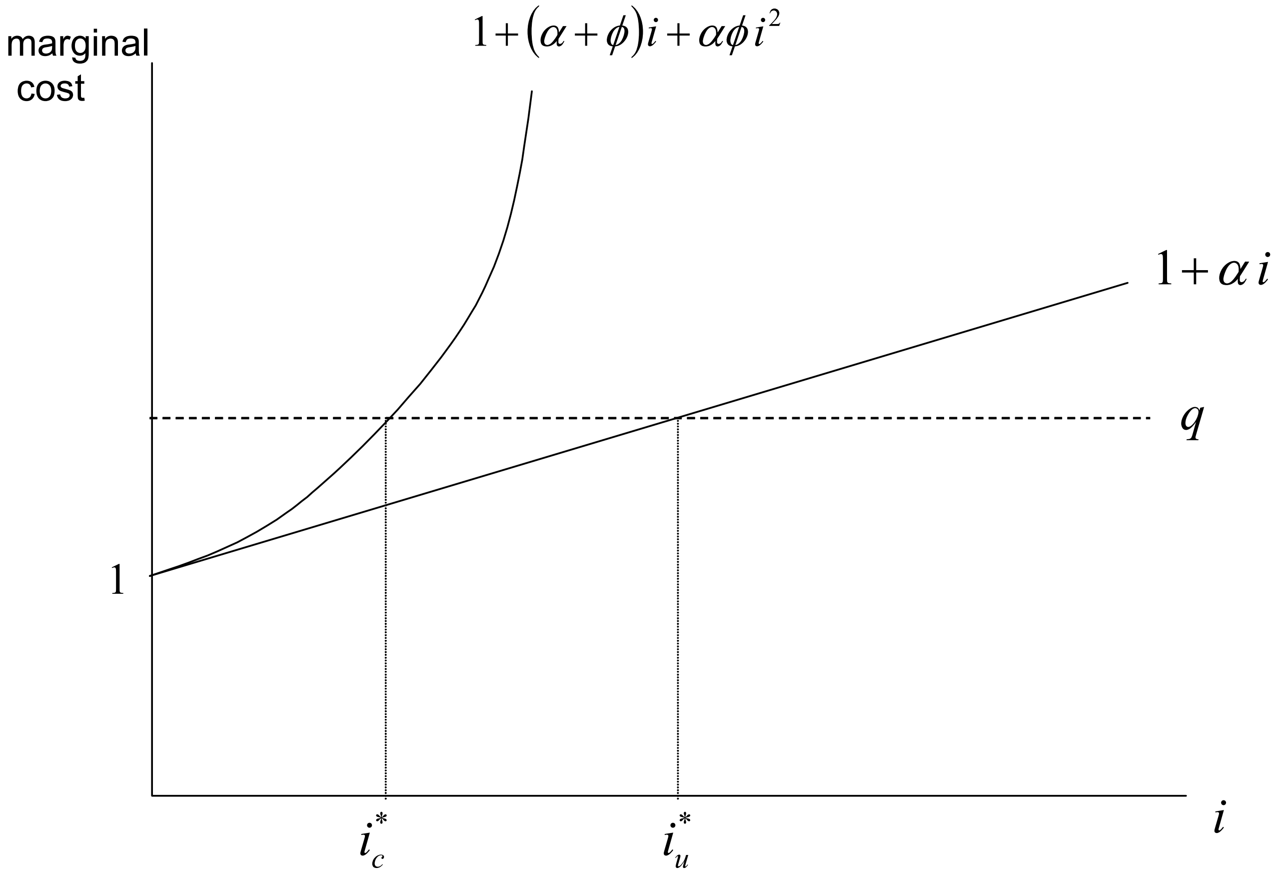

Fig. 1. Constrained and unconstrained investment. The horizontal axis measures investment, i. The vertical axis measures the marginal cost of investment. The line labeled $1+\alpha i$ measures the marginal cost of investing for an unconstrained firm, and the line labeled $1+(\alpha+\phi)i+\alpha\phi i^2$ is the marginal cost of investing for a constrained firm. The symbol q denotes a value of marginal q greater than one. The parameter $\alpha > 0$ determines curvature of the adjustment cost function, and the parameter $\phi > 0$ determines curvature of the external equity cost function. The two levels of investment, i_u^* and i_c^*, are the optimal levels for an unconstrained and a constrained firm, respectively.

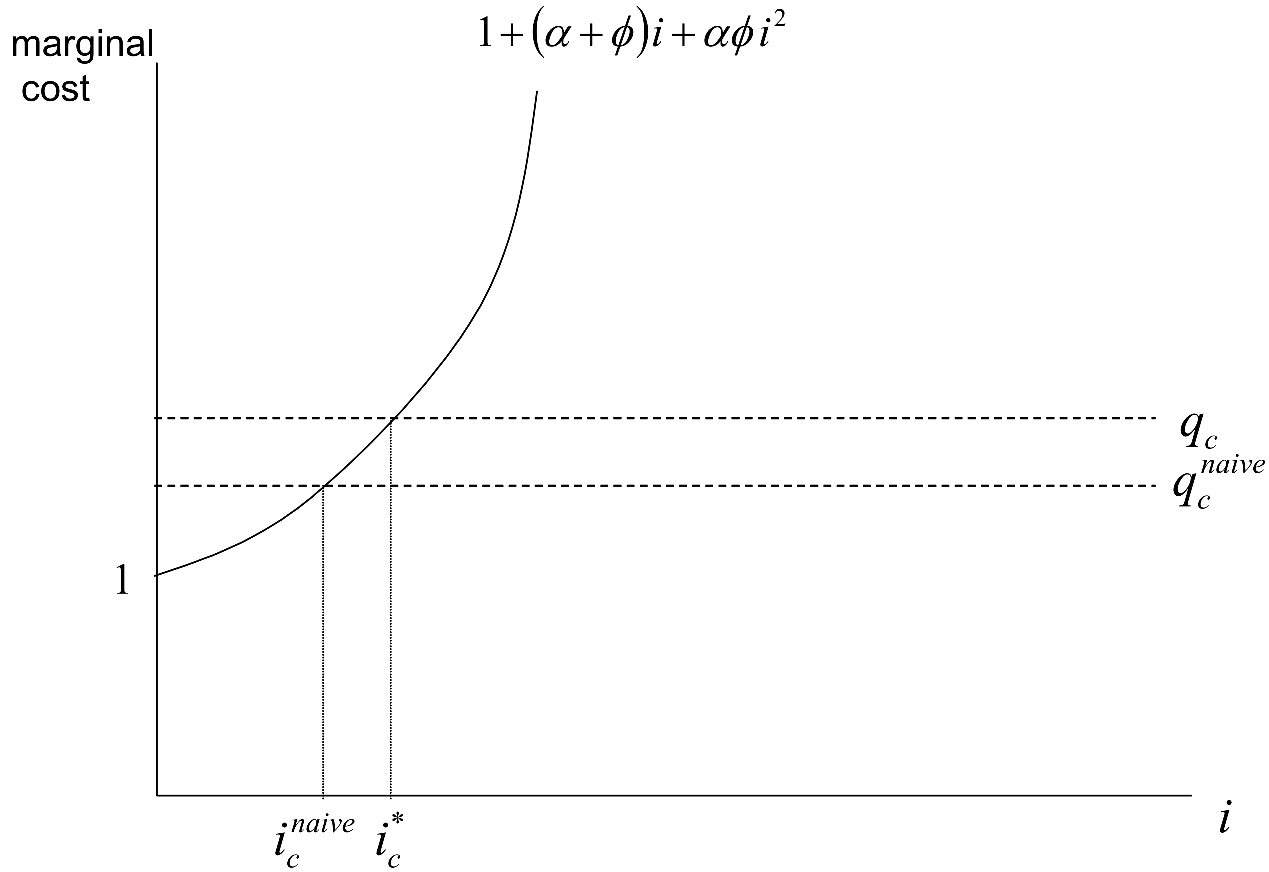

Fig. 2. Collateral constraint effects. The horizontal axis measures investment, i. The vertical axis measures the marginal cost of investment. The line labeled $1 + (\alpha + \phi)i + \alpha\phi i^2$ is the marginal cost of investing for a constrained firm. The true value of marginal q for a constrained firm is denoted as q_c, and q_c^{naive} is an estimate of the marginal q that ignores the effect of collateral on marginal q. The parameter $\alpha > 0$ determines curvature of the adjustment cost function, and the parameter $\phi > 0$ determines curvature of the external equity cost function. The optimal level of investment for a constrained firm is denoted i_c^*, and i_c^{naive} is the level of investment chosen if the collateral effect is ignored.

Table 1

Summary statistics. Calculations are based on a sample of unregulated and nonfinancial firms from the annual 2004 Compustat industrial files. The sample period is 1968 to 2003. Investment, cash flow, net debt, total debt, and equity issuance are all scaled by the capital stock. Equity issuance is the size of the issue, conditional on actually issuing. Issuance incidence is the fraction of observations with positive equity issuance. The capital stock figures are in millions of 1997 dollars. The WW index is an index of financial constraints from Whited and Wu (2006), and the KZ index is an index of financial constraints from Kaplan and Zingales (1997). Tobin's q has been removed from the KZ index. For both indices higher numbers indicate a greater likelihood both of needing external finance and facing costly external finance.

Sample	Tobin's q	Investment	Cash flow	Net debt	Total debt	Capital stock	Equity issuance	Issuance incidence
Full sample								
Mean	2.490	0.163	0.226	0.386	0.785	440.138	0.254	0.179
Median	1.121	0.120	0.193	0.311	0.469	37.649	0.076	0.000
Equity issuers								
Mean	5.874	0.240	0.269	0.102	0.849	235.371	0.254	1.000
Median	2.880	0.180	0.254	0.116	0.418	26.709	0.076	1.000
Equity nonissuers								
Mean	1.751	0.146	0.216	0.448	0.771	484.828	0.000	0.000
Median	0.943	0.112	0.186	0.339	0.479	40.584	0.000	0.000
High WW index								
Mean	2.448	0.161	0.173	0.468	0.946	20.812	0.319	0.202
Median	1.046	0.108	0.171	0.350	0.536	11.217	0.087	0.000
Low WW index								
Mean	2.531	0.164	0.279	0.304	0.624	859.482	0.169	0.156
Median	1.185	0.129	0.209	0.290	0.430	150.269	0.065	0.000
High KZ index								
Mean	1.793	0.159	0.114	0.927	1.134	336.212	0.254	0.176
Median	0.946	0.111	0.139	0.593	0.718	29.569	0.081	0.000
Low KZ index								
Mean	3.186	0.166	0.338	-0.155	0.436	544.053	0.253	0.183
Median	1.374	0.128	0.254	0.042	0.271	48.597	0.071	0.000

Table 2
Investment regressions: ordinary least squares estimates. Calculations are based on a sample of unregulated and nonfinancial firms from the annual 2004 Compustat industrial files. The sample period is 1968 to 2003. Regressions are run with both firm-specific and year-specific intercepts. Interaction term indicates the interaction of Tobin's q with the ratio of equity issuance to the capital stock. Its expected coefficient sign is negative. Overhang correction is the imputed market value of lenders' recovery claim in default normalized by the capital stock. Its expected coefficient sign is negative. The WW index is an index of financial constraints from Whited and Wu (2006), and the KZ index is an index of financial constraints from Kaplan and Zingales (1997). Tobin's q has been removed from the KZ index. For both indices higher numbers indicate a greater likelihood both of needing external finance and facing costly external finance. The expected coefficient sign on these two variables is positive. White (1980) standard errors are in parentheses under the parameter estimates. * indicates significance at the 10% level; ** indicates significance at the 5% level.

Tobin's q	Interaction term	Overhang correction	KZ	WW	Cash flow	R^2
0.0099^{**}	-0.0006^{**}	-0.2280^{**}	0.0022^{**}			0.1152
(0.0007)	(0.0003)	(0.0971)	(0.0008)			
0.0097^{**}	-0.0006^{**}	-0.1701^{**}		0.2741^{**}		0.1152
(0.0007)	(0.0003)	(0.0950)		(0.0259)		
0.0078^{**}	-0.0005^{*}	-0.2729^{**}	0.0077^{**}		0.0767^{**}	0.1508
(0.0007)	(0.0003)	(0.0975)	(0.0011)		(0.0047)	
0.0078^{**}	-0.0005^{*}	-0.1917^{**}		0.0889^{**}	0.0720^{**}	0.1396
(0.0007)	(0.0003)	(0.0958)		(0.0276)	(0.0048)	

Table 3

Investment regressions: generalized method of moments estimates. Calculations are based on a sample of unregulated and nonfinancial firms from the annual 2004 Compustat industrial files. The sample period is 1968 to 2003. The regressions include both firm-specific and year-specific intercepts, and they are estimated with the high-order moment, measurement-error consistent estimator described in Appendix B. Interaction term indicates the interaction of Tobin's q with the ratio of equity issuance to the capital stock. Its expected coefficient sign is negative. Overhang correction is the imputed market value of lenders' recovery claim in default normalized by the capital stock. Its expected coefficient sign is negative. The WW index is an index of financial constraints from Whited and Wu (2006), and the KZ index is an index of financial constraints from Kaplan and Zingales (1997). Tobin's q has been removed from the KZ index. For both indices higher numbers indicate a greater likelihood both of needing external finance and facing costly external finance. The expected coefficient sign on these two variables is positive. Fama and MacBeth standard errors are in parentheses under the parameter estimates. * indicates significance at the 10% level; ** indicates significance at the 5% level.

Tobin's q	Interaction term	Overhang correction	KZ	WW	Cash flow	R^2
0.0387**	-0.0045*	-0.7661**	0.0171**			0.4333**
(0.0114)	(0.0028)	(0.2269)	(0.0063)			(0.1313)
0.0265**	-0.0037*	-0.4052**		0.1491**		0.3559**
(0.0047)	(0.0020)	(0.1643)		(0.0569)		(0.0759)
0.0275**	-0.0019	-0.7289**	0.0167**		0.0478**	0.4585**
(0.0061)	(0.0019)	(0.1795)	(0.0023)		(0.0244)	(0.1368)
0.0202**	-0.0018*	-0.3965**		0.0782*	0.0622**	0.4670**
(0.0034)	(0.0011)	(0.1582)		(0.0404)	(0.0134)	(0.0579)

Table 4
Investment regressions: ordinary least squares and generalized method of moments estimates with alternative constraint indicators. Calculations are based on a sample of unregulated and nonfinancial firms from the annual 2004 Compustat industrial files. The sample period is 1968 to 2003. The regressions include both firm-specific and year-specific intercepts, and they are estimated with ordinary least squares and the high-order moment, measurement-error consistent estimator described in Appendix B. Interaction term indicates the interaction of Tobin's q with the ratio of equity issuance to the capital stock. Its expected coefficient sign is negative. Overhang correction is the imputed market value of lenders' recovery claim in default normalized by the capital stock. Its expected coefficient sign is negative. The WW index is an index of financial constraints from Whited and Wu (2006), and the KZ index is an index of financial constraints from Kaplan and Zingales (1997). Tobin's q has been removed from the KZ index. Cash flow has been taken out of these indices. For both indices higher numbers indicate a greater likelihood both of needing external finance and facing costly external finance. The expected coefficient sign on these two variables is positive. Fama and MacBeth standard errors are in parentheses under the parameter estimates. * indicates significance at the 10% level; ** indicates significance at the 5% level.

Estimator	Tobin's q	Interaction term	Overhang ccrrection	KZ	WW	Cash flow	R^2
OLS							
	0.0078^{**}	-0.0005^{*}	-0.2729^{**}	0.0077^{**}		0.0689^{**}	0.1542
	(0.0007)	(0.0003)	(0.0975)	(0.0011)		(0.0045)	
	0.0078^{**}	-0.0005^{*}	-0.1917^{**}		0.0889^{**}	0.0639^{**}	0.1430
	(0.0007)	(0.0003)	(0.0958)		(0.0276)	(0.0061)	
GMM							
	0.0221^{**}	-0.0015	-0.6953^{**}	0.0142^{**}		0.0537^{**}	0.4394^{**}
	(0.0037)	(0.0016)	(0.1869)	(0.0022)		(0.0137)	(0.0505)
	0.0302^{**}	-0.0048^{*}	-0.4268^{**}		0.0751^{*}	0.0420^{**}	0.4979^{**}
	(0.0075)	(0.0028)	(0.1713)		(0.0399)	(0.0140)	(0.0678)